ANDERSON

Accuracy of Electrical Metering

Electrical Engineering

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# ACCURACY OF ELECTRICAL METERING

BY

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THESIS

Submitted in Partial Fulfillment of the Requirements for the

Degree of

MASTER OF SCIENCE

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May 31, 1912

I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

ARVID ROBERT ANDERSON

ENTITLED ACCURACY OF ELECTRICAL METERING

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

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# ACCURACY OF ELECTRICAL METERING

## INTRODUCTION

In all the literature of Electrical Engineering, there is perhaps no subject of equal importance that has received less attention than the problem of electrical metering. What the reason for this may be is not at once apparent, for surely the significance of this problem must have been constantly recognized by the profession. Perhaps the extensive and everchanging nature of the subject has so hidden the logical point of attack as to discourage any attempts at a complete treatment. And indeed, a complete treatment would be almost impossible, for the requirements of a meter can be embodied in mechanical combinations almost without end, and it is doubtful if any one living man knows of all that have been tried.

It is needless, therefore, to say that in the following paper no such complete treatment will be attempted. Even if time and space were afforded the value of the result would be doubtful, except, perhaps, as an interesting bit of history, or a study in human ingenuity. To make the paper of most value and greatest interest to the present day engineer, therefore, the discussion will be confined to the most representative instruments of modern practice,--those which experience has shown to be the best adapted. Nor will the complete field of such instruments be covered, but only those connected with power measurement will be studied as it is believed that

these are of greatest significance in the extended application of electricity.

Before great progress is possible in any science and before its practical application becomes a possibility, it is necessary to deal with it not only in a qualitative but in a quantitative way. It is hardly an exaggeration to say that the foundation of all science lies in the "yard stick". To be sure, a great deal may be learned of the nature of forces acting about us by the mere observation of recurring phenomena, and indeed from the qualitative study of these may be deduced very important general principles and laws: but before any number of such observations can be built up in logical order and made to support one another it becomes imperative to determine the magnitude of the quantities involved. In other words, in order that a science may be built up and progress, units of measurement must be extablished, and means for their "enumeration" devised.

The fundamental units are those of length, mass, and time, and from these have come many derived units to suit the particular needs of each science. In electrical engineering many such derived units are employed, and upon their accurate measurement much depends. The volt, the ampere, the kilowatt, and the kilowatt-hour are the best known of these units, and of them the kilowatt-hour is of particular interest since it is a gage of dollars and cents. It is in these units that the most extensively used electrical meters are calibrated, and which meters it is the purpose of this

paper to present.

There are other electrical meters of considerable importance and interest, such as the frequency meter, the power-factor indicator, the synchroscope, the ohmmeter, and others, but these are used on a smaller scale, and are generally confined to the laboratory or station switchboard. As confinement within certain limits is a matter of necessity in dealing with this subject, the discussion will be limited to instruments intimately associated with the measurement of electrical energy, namely, the ammeter, the voltmeter, the wattmeter, and instrument transformers. These instruments will be taken up in the order given above, and studied individually to point out in a general way the most approved method of construction, and the best arrangement of circuits and compensating devices. Such mathematical discussion will be introduced as will serve to make clear the operation of the instrument, indicate the probable limits of accuracy, and show how errors may be corrected for by consideration of the constants of the instrument. A brief discussion of typical circuits containing more than one of these instruments will follow, the best arrangements under given conditions indicated, and the extent to which the accuracy of the instrument reading is affected by the position of the instrument in the circuit pointed out. In conclusion a brief summary and such general results as seem warranted and of sufficient importance will be given.

# CHAPTER I

## FUNDAMENTAL PRINCIPLES OF ELECTRIC METERS

Wherever an electrical magnitude is to be measured, the expenditure of energy in the measuring device is an unavoidable consequence, and wherever an electrical magnitude is to be measured by a direct reading instrument a dynamic force is required. If a piece of apparatus can be constructed so as to convert electrical energy into some other form, and that energy measured, then this piece of apparatus connected into the same circuit with a larger piece of apparatus in such a way that the energy transformed by the first piece of apparatus is a predetermined function of the energy consumed by the second piece of apparatus, then the first acts as a meter to the second which is called the load. If instead of actual work to be measured the first piece of apparatus can be made to exert a force which is a predetermined function of the power consumed by the second piece of apparatus the same purpose is served, and the first becomes a meter for the second. So it is seen that the essential requirements for a meter are really quite simple, and it is possible to embody these principles in an almost infinite number of forms. The real problem is how to make the most efficient, most reliable, most durable, simplest, and least expensive instrument.

The four properties of the electric current that have been chiefly utilized for the purpose of metering are the electromagnetic, electrothermal, electrostatic, and

electrolytic. Of these, the first is by far the most important and extensively used at the present time, and it is with meters of this type that it is purposed to deal in this paper.

At one time the electrolytic meter, based on Faraday's law for the deposition of metal by an electric current passed thru a solution of a salt of the metal, was extensively used to measure ampere hours at a constant voltage, and thus the power; but with the developement of electromagnetic meters and the advent of A. C. distribution these meters, having nothing in particular to recommend them, and being inoperative on A. C., went rapidly out of use. The electrostatic meter is based on the attraction between two charged discs. For discs of a convenient size, at normal potentials, this attraction is quite small, requiring exceedingly delicate construction and making an instrument operating on this principle very sensitive and erratic. An instrument of this type has but very little to recommend it except where the potential is high, and where it is important that the instrument consume the minimum of power. The electrothermal or more definitely the heating effect of the electric current when passing thru a resistance is made use of in the so-called hot wire instruments. A wire made taut between two points by a spring elongates as the wire is heated up by the current passing thru it, and in so doing deflects a pointer attached to it thru a system of wires and springs that multiply the motion. An instrument of this type serves equally well

on A. C. and D. C., but it is difficult to make it reliable, that is to retain its calibration, it is not very substantial, and very likely to be damaged by slight overload. Moreover, it is not very efficient, as it requires generally between one tenth and two tenths of an ampere to operate. On the whole, then, these instruments have failed to give the satisfaction that can be obtained by other types, and so have gone out of use except where they are advantageous for some special work such as for the measuring of high frequency currents.

There remains, then, the electromagnetic type of instrument, operating on the principle of magnetic attraction and repulsion. This principle has shown itself so well adapted to metering instrument in point of reliability and durability that practically all commercial instruments of today, with the possible exception of a few high potential electrostatic voltmeters, are of this type. The possibility of applying this principle in almost an infinite number of forms has resulted in the production of a great number of meters of widely varying types operating on this principle. The fundamental essential of such an instrument is the arrangement of conductors in such a way that when a current is passed thru them two magnetic fields are set up which react upon each other and produce a force or torque. A magnetic substance, such as iron, may or may not be used in either one or both of the fields, and the two fields may be induced by one coil alone or by two coils, or one field may be permanent. Furthermore, the fields may be so arranged that the action between them is a

direct force, or a torque; and this force, or torque, may be balanced by a counter-weight or spring in any way that is most convenient. This serves to indicate the variety of forms of instruments of this type, but the principle of operation is the same for all of them namely, the movement of a point over a calibrated distance due to the magnetic force set up by an electric current in one or more coils until this magnetic force is counterbalanced by an opposing force.

At this point it may be well to consider the magnetic field due to a coil of wire carrying a current, and the force exerted between two such fields. With a core of constant permeability, such as air, the flux varies directly as the current flowing in the coil, or

$$\varphi_1 = K_1 I$$

With a core of varying permeability, such as iron, the flux depends not only upon the current but also upon the permeability, or

$$\varphi_2 = K_2 \mu I$$

If, for a given value of I there existed a definite value of $\mu$, then $\mu$ would be a univalent function of I, and the above expression might be written $\varphi_2 = K f(I)$. But it is known that, due to the hysteresis of iron, this is not the case; that for a given value of current $\mu$ may have any value within certain limits depending upon the previous condition of the iron; and that, therefore, the flux is not a univalent function of the current. The attraction between two solenoids

in given relative positions varies as the product of the fluxes they set up. If the solenoids are coaxial the force exerted tends to produce only a motion of translation, if the axes of the solenoids are at right angles the tendency is to produce only rotation, and for any intermediate position there is the tendency to produce both rotation and translation unless the coils are absolutely concentric as regards their magnetic fields. Then the action of one coil upon the other will always be a turning moment or torque.

Any subdivision or classification of electromagnetic meters in regard to their construction or operation must necessarily be somewhat superficial and crude. At present the production of torque in a meter, and its variation with current are the points of interest and for this reason it is believed that the following subdivision will prove of service.

Case 1. Both magnetic fields contain iron, and each is induced by a separate coil thru which current is passed. In this case, if $\varphi_1 = K_1 \mu_1 I_1$ and $\varphi_2 = K_2 \mu_2 I_2$, the force F exerted between the two coils in given relative positions is

$$F = \varphi_1 \varphi_2 = K_3 \mu_1 \mu_2 I_1 I_2 \qquad (1)$$

Case 2. Both magnetic fields contain iron, and both are induced by one and the same coil. In this case, if $\varphi_1 = K_1 \mu_1 I$ and $\varphi_2 = K_2 \mu_2 I$, the force F exerted between the two fields in relative positions is

$$F = \varphi_1 \varphi_2 = K_3 \mu_1 \mu_2 I^2 \qquad (2)$$

Case 3. One magnetic field contains iron, and the other air. Each field is induced by a separate coil. In this case, if $\varphi_1 = K_1 \mu_1 I_1$ and $\varphi_2 = K_2 I_2$,

$$F = \varphi_1 \varphi_2 = K_3 \mu_1 I_1 I_2 \qquad (3)$$

Case 4. One magnetic field contains iron, and the other only air; and one coil alone induces both fields. In this case, if $\varphi_1 = K_1 \mu_1 I$ and $\varphi_2 = K_2 I$,

$$F = \varphi_1 \varphi_2 = K_3 \mu_1 I^2 \qquad (4)$$

Case 5. Both magnetic fields are of air, and separate coils induce them. In this case, if $\varphi_1 = K_1 I$, and $\varphi_2 = K_2 I_2$

$$F = \varphi_1 \varphi_2 = K_3 I_1 I_2 \qquad (5)$$

Case 6. One magnetic field is permanent, and the other induced by current in a coil containes iron. If $\varphi_1 = K_1$ and $\varphi_2 = K_2 \mu_2 I$

$$F = \varphi_1 \varphi_2 = K_3 \mu_2 I \qquad (6)$$

Case 7. One magnetic field is permanent, and the other, induced by current in a coil, contains no magnetic substance. If $\varphi_1 = K_1$ and $\varphi_2 = K_2 I$

$$F = \varphi_1 \varphi_2 = K_3 I \qquad (7)$$

Case 8. This case applies to alternating current meters only. An alternating current is passed thru one coil, generally on an iron core, and the alternating field set up induces E. M. F.s in an armature of non-magnetic material, which E. M. F.s set up eddy currents, and which eddy currents set up a magnetic field which reacts upon the field mentioned or primary magnetic field. Two primary fields displaced in time so as to give a rotary resultant field and thus continuous torque may be and generally are used in induction meters. The expression for torque or force in this case is not obtained as readily as those given above, but the secondary flux under given conditions will vary about as the primary flux, and if a single current is considered as entering the meter, $\varphi_1 = K_1 \mu_1 I_1$ and $\varphi_2 = K_2 \mu_1 I_1$. Whence

$$F = \varphi_1 \varphi_2 = K_3 \mu_1^2 I_1^2 \qquad (8)$$

If the rotating primary field is produced by two currents displaced in time the expression for torque is not so readily obtained and will not be given here.

It is seen that these eight cases cover practically all the combinations that could be used to produce an electro-magnetic instrument, and that the expression for force or torque is different in each case. In case 1, 3 and 5 the product of two different currents enters into the expression for torque, and, therefore, these combinations might be used for the measurement of power. In cases 1 and 3, however, the permeability also enters, and whether the combination is used for D. C. or A. C., this would be a serious source of trouble, as will be explained more fully later, and would make these cases entirely inadequate for the measurement of power. Case 5, then, represents the only combination that is available for the measurement of power, and is the one that is used. In these three cases there is no reason why $I_1$ and $I_2$ might not be made equal, and in this event all of the eight cases might be used for the measurement of current,--the first five for either A. C. or D. C., 6 and 7 for D. C. only, and 8 for A. C. only. In six out of eight of these cases the permeability $\mu$ enters, and it will be well, therefore, before going further to consider this factor with reference to direct current and alternating current.

Fig. 1 represents a hysteresis loop of the iron under consideration, the maximum point being determined by the maximum current taken by the particular instrument. To be brief, if the instrument is used for D. C., then for a given value of current, i, the permeability may have any value between the limits B and C. For example, if the iron

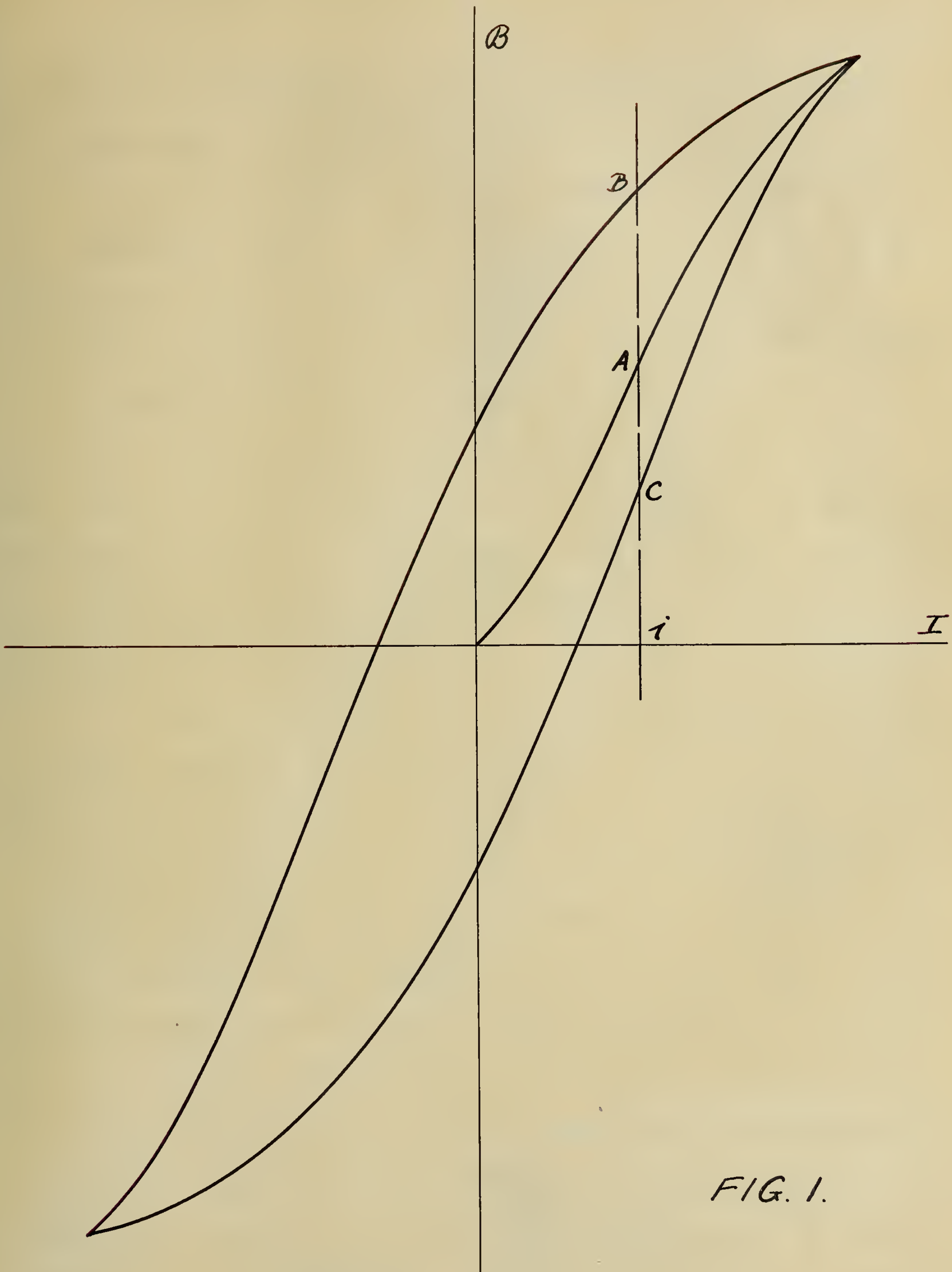



FIG. 1.

were entirely demagnetized before passing a current thru the instrument the permeability would be as represented at point A; if the maximum value of current were passed thru the instrument and the value then decreased to i the permeability would be as represented by point B; and if the maximum current were passed thru the instrument in the opposite direction, and then the value of current equal to i reached, the permeability would be as represented by C. This serves to show how the use of iron subject to varying flux densities in D. C. instruments has a tendency to make the indications of the instrument erratic, and for this reason is not to be recommended.

The use of iron in A. C. instruments will be considered. Suppose, as a starting point, that a sine wave of current flows through the instrument, as represented by the full line in Fig. 2, and that the corresponding hysteresis loop for each cycle is as represented by the full line loop in the same Fig. This wave has a r.m.s. value equal to the maximum value divided by the square root of two which it is desired to read; and for all sine waves of current having the same effective value the iron goes thru the same hysteresis cycle. So that, for a sine wave of given effective value, the effective permeability of the iron may be considered as a definite value, and the permeability of the iron may, therefore, be considered as a univalent function of the current. But let a very greatly distorted current wave having, however, the same effective value as the sine wave,

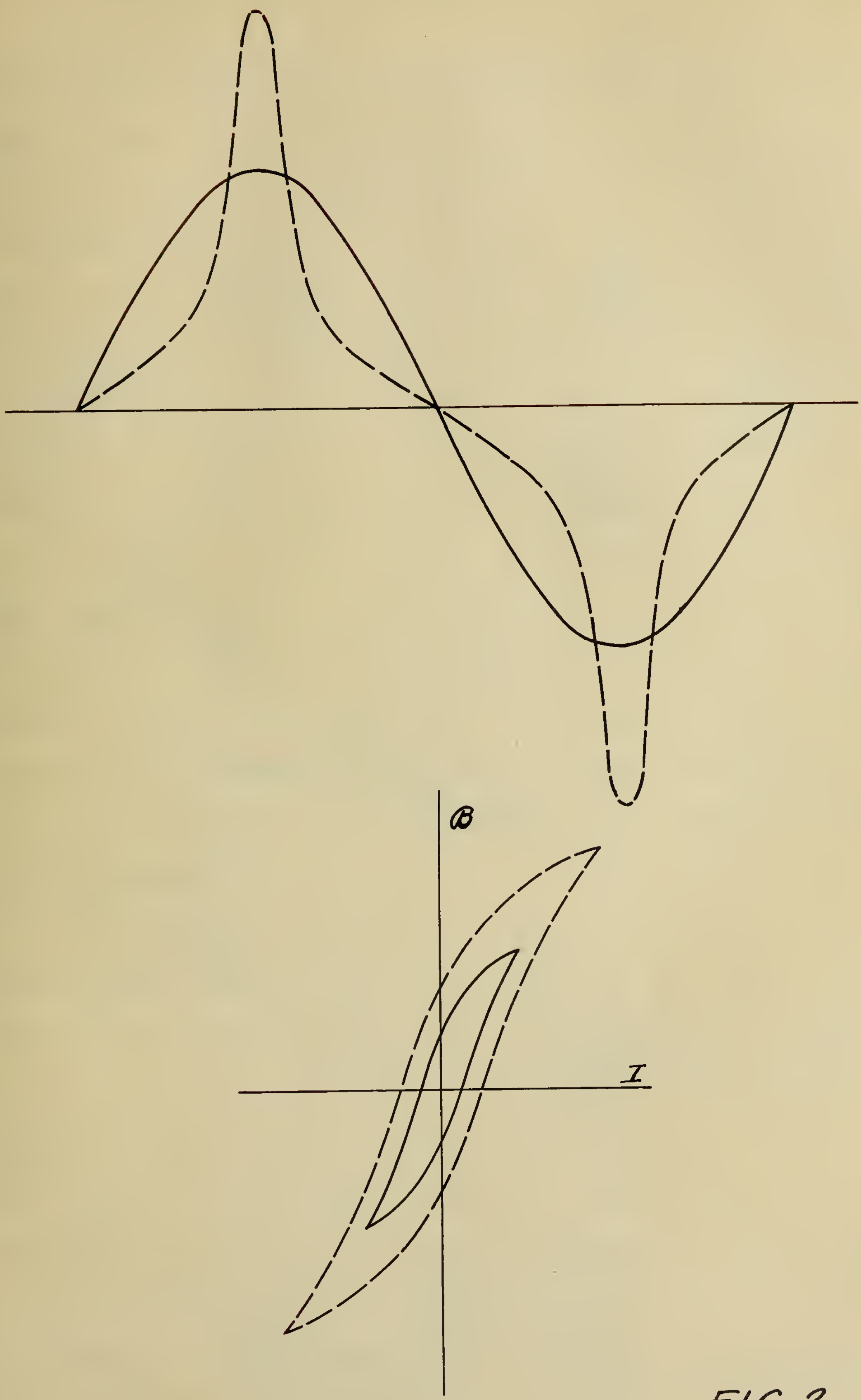



FIG. 2

pass thru the instrument. The dotted line in Fig. 2 represents such a wave. It is seen that the maximum value is greater than that of the sine wave, and therefore the hysteresis cycle for this wave will be as represented by the dotted cycle, giving a different effective value of permeability for the same effective value of current. Generally, in practice the approximation to a sine wave is quite close, and the effective value of permeability is not greatly effected by ordinary distortions of the current wave. It should be born in mind, however, that theoretically at least wave shape does affect the reading of instruments containing iron in their magnetic fields, and that for unusual wave distortion, error in the instrument reading from this source may become very considerable. In what follows a sine wave of current will be assumed, when the permeability will be a univalent function of I.

Although iron is still used in some D. C. swtichboard meters, it is evident from what has been said that its use renders the instrument unreliable for accurate measurements. For this reason it is not used in instruments designed to give a reasonable degree of accuracy, and, therefore, as far as D. C. measurement is concerned the first four cases as well as the sixth may be dropped now. There remains then the fifth and seventh cases available for D. C. measurement, and more will be said later of the relative merits of these two for this purpose.

For the measurements of an alternating current any

one of the first five cases is applicable as well as the eighth, the desirability of one over another depending somewhat upon the particular conditions. For current measurements cases 2 and 4 have found perhaps the widest application; for voltage measurements case 5 as well as 2 and 4 has been extensively used; and case 8 has been somewhat used for current measurements; but has been very extensively used, in the elaborated form of the integrating induction meter, for the measurement of electrical energy.

For the measurement of electrical power, both D. C. and A. C., case 5 is the only one that is adequate, because the measurement of power requires that the torque shall be a univalent function of the product of in phase currents, and the introduction of the permeability factors in cases 1 and 3 destroys this condition. Case 5 is the most general case of all, being applicable to current, potential, power, and energy measurements on both D. C. and A. C., and for this reason it may be well to consider briefly the two classic forms in which this principle has been embodied, namely, Lord Kelvin's balance and Siemen's electrodynamometer.

Ford Kelvin's balance, as illustrated in Fig. 3, consists of four stationary coils and two coils fixed at either end of a pivoted beam. These coils are so connected as to give them polarity upon passing a current thru them which will cause deflection of the beam in one direction. The beam is then brought back to the zero position, as indicated by the pointer, by a rider on a graduate scale. When

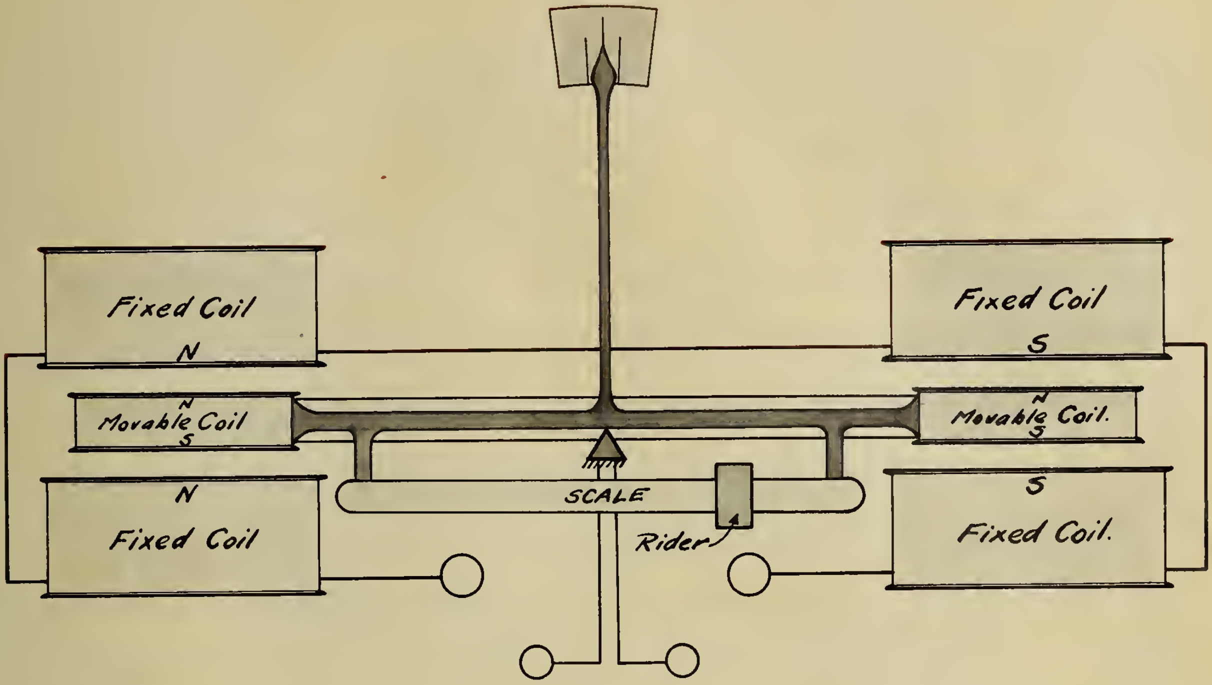

Fixed Coil
N
Movable Coil
N
S
N
Fixed Coil
Fixed Coil
S
Movable Coil.
N
S
S
Fixed Coil.
SCALE
Rider

FIG. 3

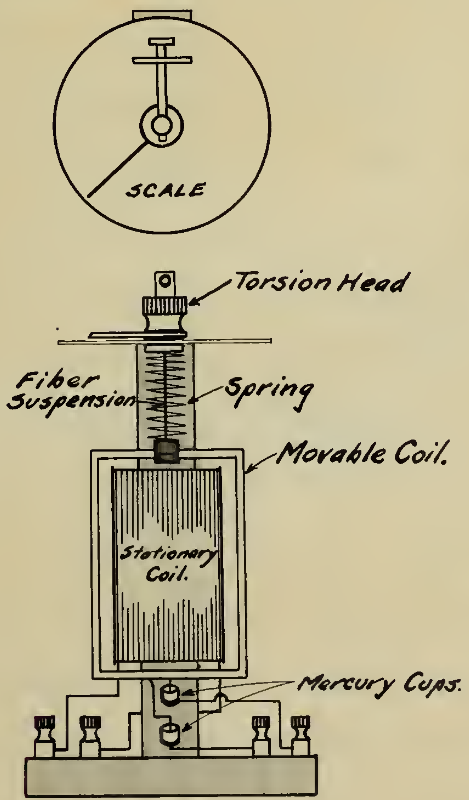

SCALE
Torsion Head
Fiber Suspension
Spring
Movable Coil.
Stationary Coil.
Mercury Cups.

FIG. 4

equilibrium is established the moment or torque of the rider equals the magnetic torque, and thus becomes a measure of it. Since at the time of reading the coils are always in the same relative positions, equation (5) holds, and the distance of the rider from the fulcrum measured horizontally varies directly as the product of the currents in the stationary coils and the moving coils.

In Fig. 4 is represented a Siemens' dynamometer, having a stationary coil and a moving coil, the magnetic fields of which are concentric and at right angles. The moving coil is suspended by a fibre and connected to a spiral spring one end of which is fixed to a knurled head carrying a pointer which may be turned so as to give any desired degree of torsion in the spring. Current is lead into and out of the moving coil thru mercury cups into which the terminals of the moving coil dip. When current is passed thru the coils a torque is exerted that rotates the moving coil, but by turning the torsion head sufficient counter torque is supplied to bring the moving coil back to its zero position as indicated by a pointer attached to the moving coil. A pointer on the torsion head indicates on a circular scale the angle thru which it has been turned, and since by Hook's law the resisting torque of the spring it directly proportional to the angular displacement, this indication is a measure of the torque supplied to balance the magnetic torque. Here, again, when readings are taken the relative positions of the two coils are always the same, and hence equation (5)

holds. When equilibrium is reached the torque of the spring equals the magnetic torque, or,

$$K_1 \, \theta = K_2 \, I_1 \, I_2$$

Whence $$\theta = K \, I_1 \, I_2 \qquad (9)$$

Siemens' dynamometer may be readily modified to make a direct reading instrument by fixing the torsion head and noting the deflection of the moving coil. A convenient laboratory form is made by providing a metal fibre suspension for the moving coil and attaching to the lower end of the coil a spiral spring, one end of which is fixed. The suspension and spiral spring serve as leads for the passage of current thru the moving coil. When current is passed thru the instrument the moving coil is deflected until equilibrium is reached at a point where the increasing torque of the spring becomes equal to the decreasing magnetic torque. A mirror attached to the moving element indicates the amount of this deflection by the reflection of a beam of light on a graduated scale about a meter distance. It is evident that with a given current the torque for all positions of the moving coil is not the same, but is a maximum when the coils are at right angles, and zero when they are parallel, whereas the resisting torque of the spring is directly proportional to the angular displacement. The proportionallity established for the Siemens' dynamometer with the torsion head will not, therefore, hold in this case, and unless this variation in

magnetic torque with position can be compensated for in the shape of the scale, no proportionality between product of currents and scale deflections will exist, and a calibration of the instrument over its entire range will be necessary. It has been found that a straight scale placed parallel to the fixed field is the most satisfactory, and that for reasonable deflections the readings on such a scale will be practically proportional to the product of currents. If $I_1 I_2$ represents the product of current, and d represents the deflection on such a scale, then the variation of $I_1 I_2$ with d may be found as follows:

Let the moving coil be deflected thru an angle $\theta$ as shown in Fig. 5. The distance from the scale to the mirror is a, and the deflection on the scale is d. The maximum torque, in the zero position, is $T_0 = K_1 I_1 I_2$, and if it be taken to vary as the cosine of the angle of displacement, the torque in any position is

$$T = K_1 I_1 I_2 \cos \theta \qquad (10)$$

The resisting torque R, of the spring varies directly as $\theta$, or

$$R = K_2 \theta \qquad (11)$$

When equilibrium is reached, T = R, or

$$K_1 I_1 I_2 \cos \theta = K_2 \theta$$

and

$$I_1 I_2 = \frac{K_3 \theta}{\cos \theta} \qquad (12)$$

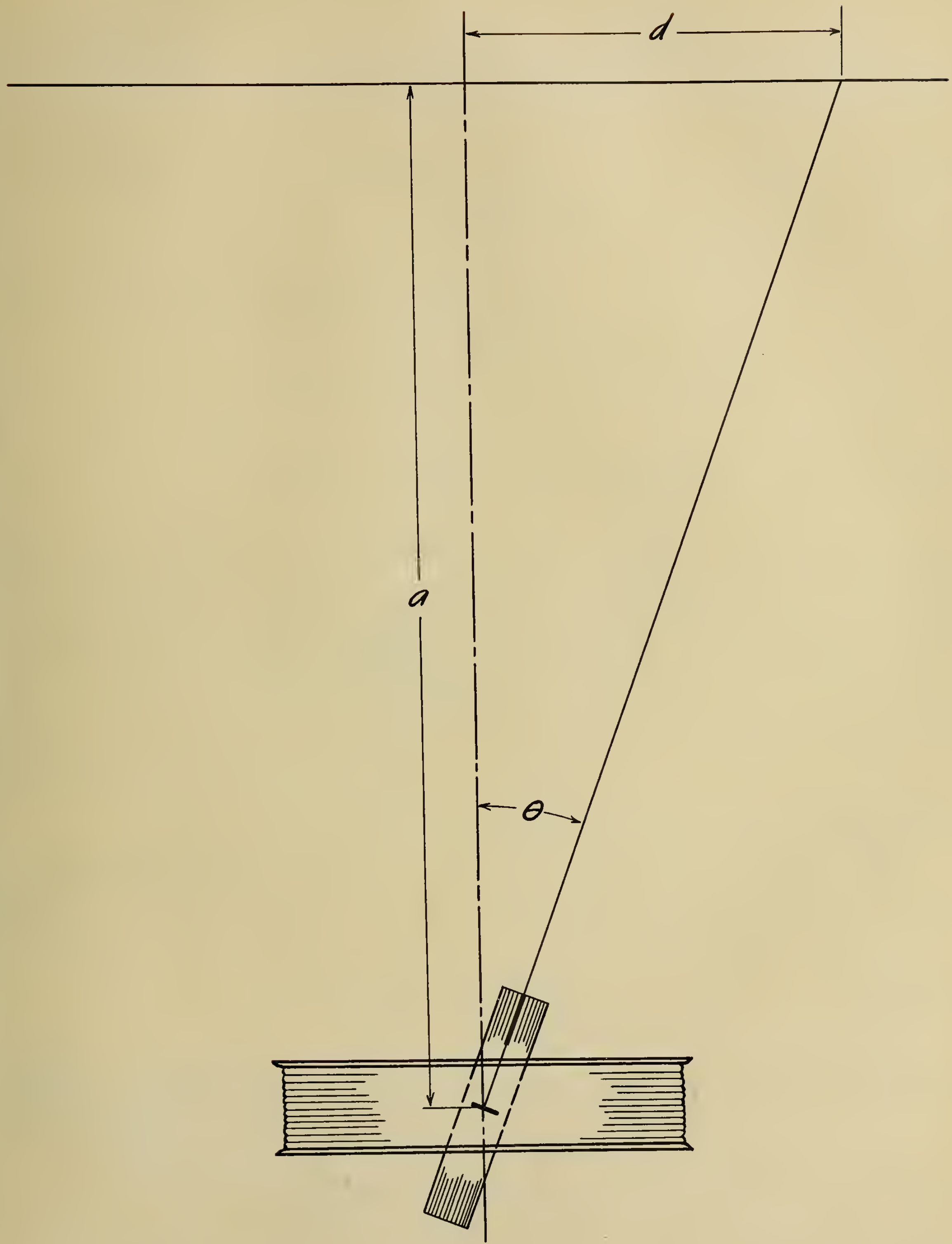



FIG. 5

From Fig. 5,

$$\theta = \text{arc tan } \frac{d}{a} \tag{13}$$

and

$$\cos \theta = \frac{a}{\sqrt{a^2 + d^2}} \tag{14}$$

Substituting (13) and (14) in (12)

$$I_1 \ I_2 = \frac{K_3}{a} \sqrt{a^2 + d^2} \text{ arc tan } \frac{d}{a}$$

$$I_1 \ I_2 = K \sqrt{a^2 + d^2} \text{ arc tan } \frac{d}{a} \tag{15}$$

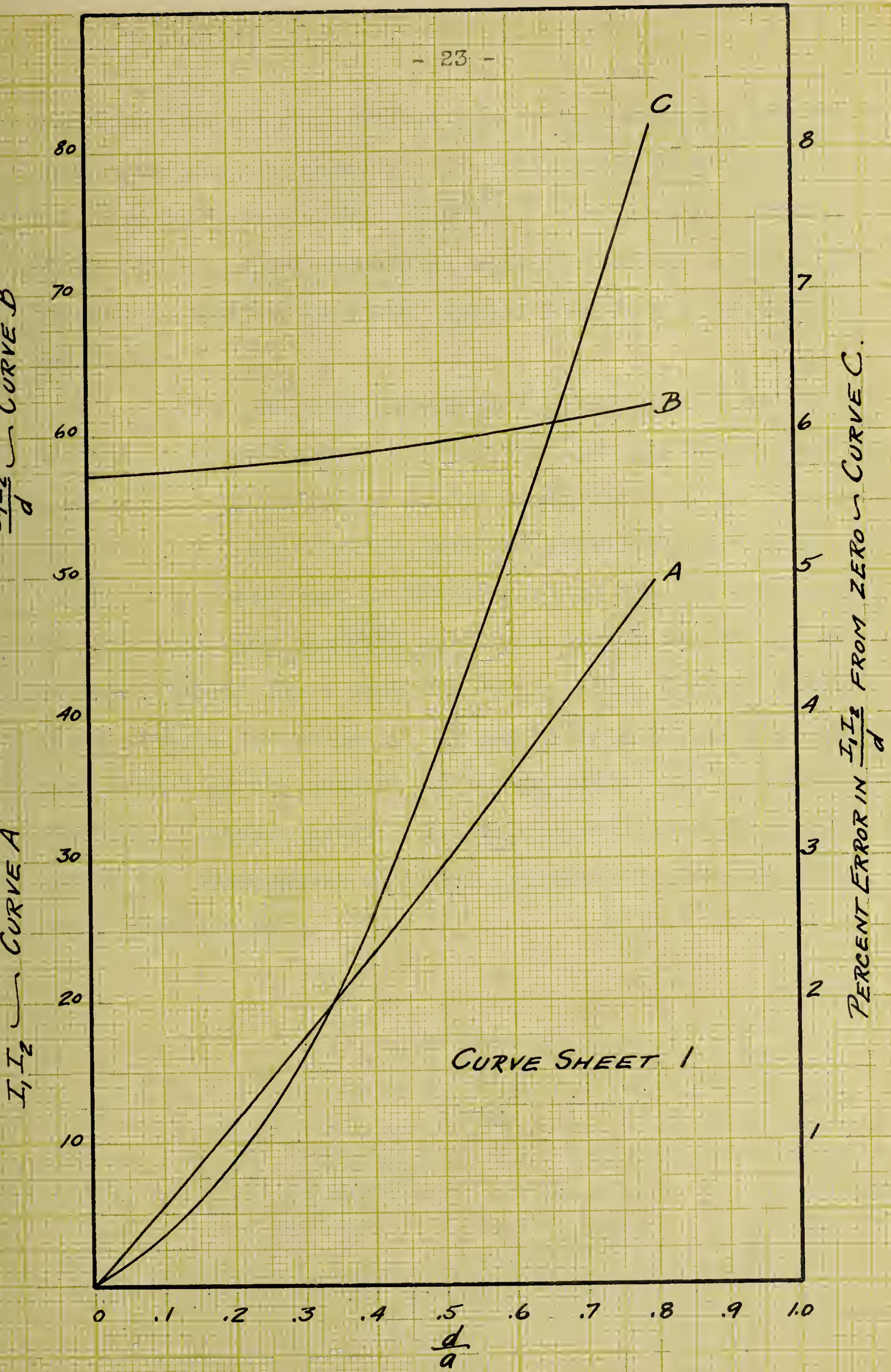
On curve sheet 1, Curve A represents equation (15), assuming K and a equal to unity; and Curve B represents the variation of the proportionality factor $\frac{I_1 \ I_2}{d}$ with d; and Curve C represents the percent error introduced by assuming proportional variation of $I_1 \ I_2$ with d.

If a circular scale is used instead of a straight scale the problem may be worked out in the same way, and similar curves drawn. In this case the expression obtained is

$$I_1 \ I_2 = K \frac{d}{\cos a \ d} \tag{16}$$

and the percent error increases much more rapidly with the deflection than in the case of a straight scale.

If this deflecting dynamometer be further modified by substituting for the fibre suspension pivoted jewel bearings and spiral springs, the instrument will have taken on the essentials of a portable wattmeter. Of course, the relations given above generally cease with the modifications


C
B
A
80
70
60
50
40
30
20
10
8
7
6
5
4
3
2
1
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
$\frac{d}{a}$
$\frac{I_1 I_2}{d}$ — CURVE B
$I_1 I_2$ — CURVE A
PERCENT ERROR IN $\frac{I_1 I_2}{d}$ FROM ZERO — CURVE C.
CURVE SHEET 1

introduced in such a portable meter, but the scale may be calibrated and the necessity of knowing the mathematical relation done away with. However, with the coil arrangements found in some commercial portable meters the relation between $I_1$ $I_2$ and d is quite readily determined, as will be seen later.

This electrodynamometer type of instrument is, perhaps, adapted to more general use than any other electrical instrument. It is suited to the measurement of A. C. as well as D. C., and when so used the instantaneous value of torque is proportional to the product of the instantaneous values of current, and the resultant torque is the average of these values for a complete cycle. Furthermore, since it is made up of two coils, it may be made to measure either one current or the product of two currents, and is therefore adapted to measure power. A more complete discussion of the use of the dynamometer for this purpose will be given later under "Wattmeters".

# CHAPTER II

## AMMETERS AND VOLTMETERS

An ammeter is an instrument used for the measurement of current. So is, essentially, a voltmeter. For this reason it seems advisable to consider them together. The difference between the two is chiefly that ammeters are called upon to measure current over a very large range, whereas voltmeters measure only very small currents. In fact, a voltmeter may be considered as a milli-ammeter, calibrated by the application of Ohm's law to read volts directly, for if the resistance of the instrument remains constant the current flowing thru it will be directly proportional to the E. M. F. impressed across it. But the resistance of the instrument must remain constant. It is well known that the resistance of most metals, including copper, varies with the temperature. Hence, if the voltmeter and circuit be of copper the above condition of constant resistance cannot be realized with variations in temperature. It is customary, therefore, to make the coil of copper but of low enough resistance so that a comparatively high resistance of some such alloy as manganin, having a negligible temperature coefficient, may be connected in series with it. This reduces the error to temperature changes in proportion as the percent manganin is increased.

For convenience in presentation, these instruments will be divided into three classes, viz, those operating on

D. C. only, those operating on either D. C. or A. C., and those operating on A. C. only.

The only principles considered in the previous chapter that cannot be made to give an average unidirectional torque on A. C. are those involving permanent magnets, namely cases 6 and 7, and hence these may be used for the measurement of direct current only. Case 6 contains iron in the varying magnetic field, and for this reason as has been demonstrated is not desirable. This leaves only case 7 to be considered, but first it will be interesting to note an example that falls, perhaps, somewhere between these two cases.

Referring to Fig. 6, a small soft iron armature s - n is pivoted in the field of the permanent magnet N - S, and by magnetic induction takes up the position shown by the full line when no current flows thru the coils C - C which are at right angles to the lines of force of the permanent magnets. When a current flows thru these coils a component field is introduced, and the direction of the resultant field is different from the original field. This causes the iron armature to take up a position indicated by the dotted lines, and the deflection of the pointer is read on a scale. Not only does the direction of the field change upon passing a current thru the coils, but also the strength of the field changes, with a consequent change in the magnetic induction in the iron. But in this case the torque is not exerted against a spring, it is exerted merely to swing the armature

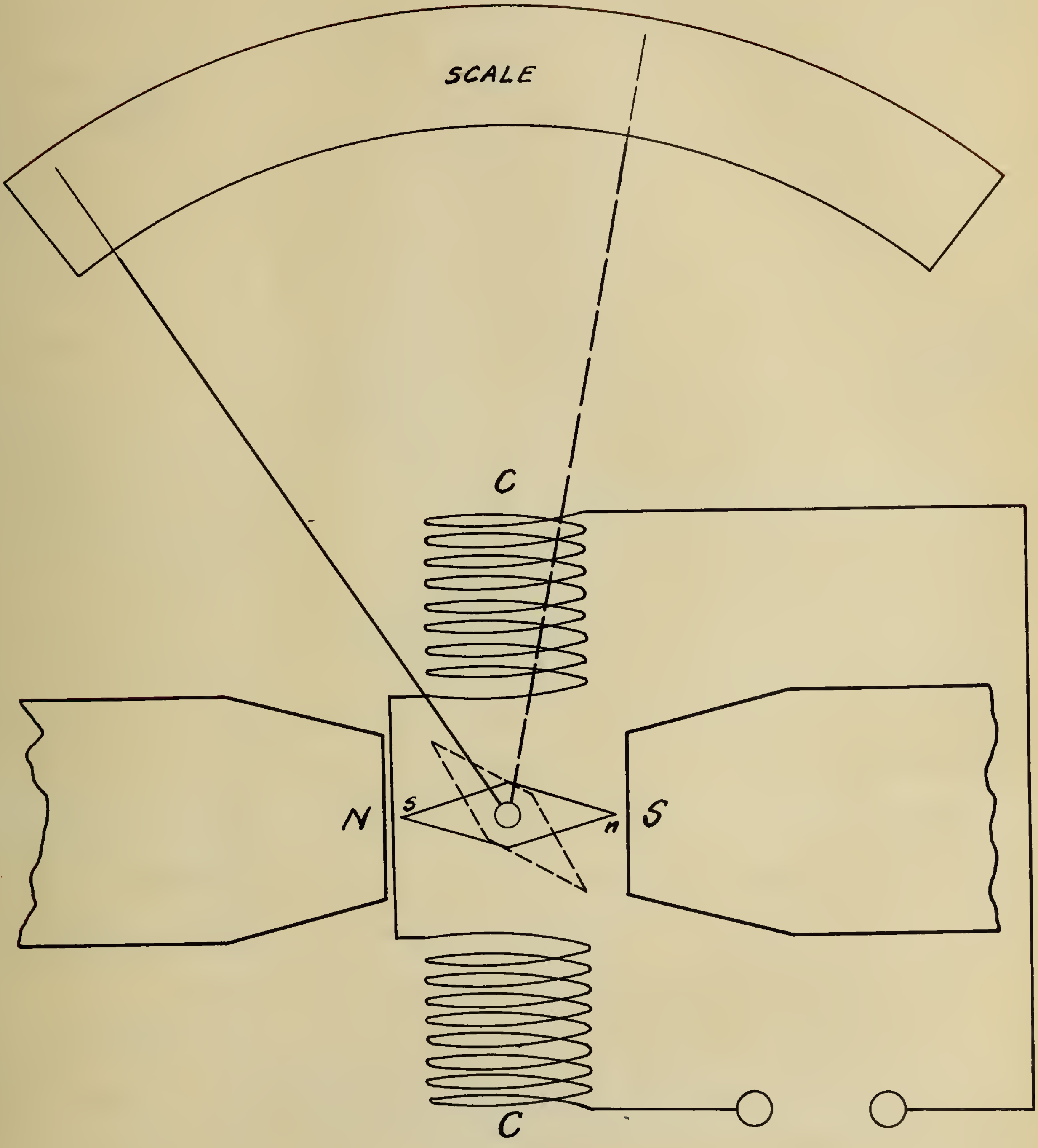



FIG. 6.

parallel with the lines of force of the resultant field, and hence the value of torque is not vital. The variation in magnetic induction for a given magnetizing force is of consequency only in as much as it influences the direction of the resultant field, and this influence is very slight. This arrangement is quite simple, and is perhaps the most reliable that can be obtained using iron. It is, however, quite sensitive to vibrations, and requires considerable time to come to rest. Furthermore, the permanent magnets, subject to the influence of the comparatively strong field of the coils, are likely to deteriorate and throw the instrument out of calibration. As an ammeter it has the advantage of taking all the current thru its windings instead of shunting only a portion of the total current as is the most common practice at present. The advantage of this will become more evident later when the effects of temperature upon shunted instruments is discussed. Wound with many turns of fine wire, this arrangement becomes a voltmeter.

The principle given under case 7 is the one that has found most general application in D. C. measuring instruments, and the best form seems to be the movable coil and fixed magnet type, being essentially a portable D'Arsonval galvanometer, with jewel bearings and a pointer attached to the moving coil giving indication on a calibrated scale.

Fig. 7 shows by a simple sketch the arrangement of essential parts in the Weston instrument. The movable coil is wound with fine copper wire, requiring but a fraction of a volt to

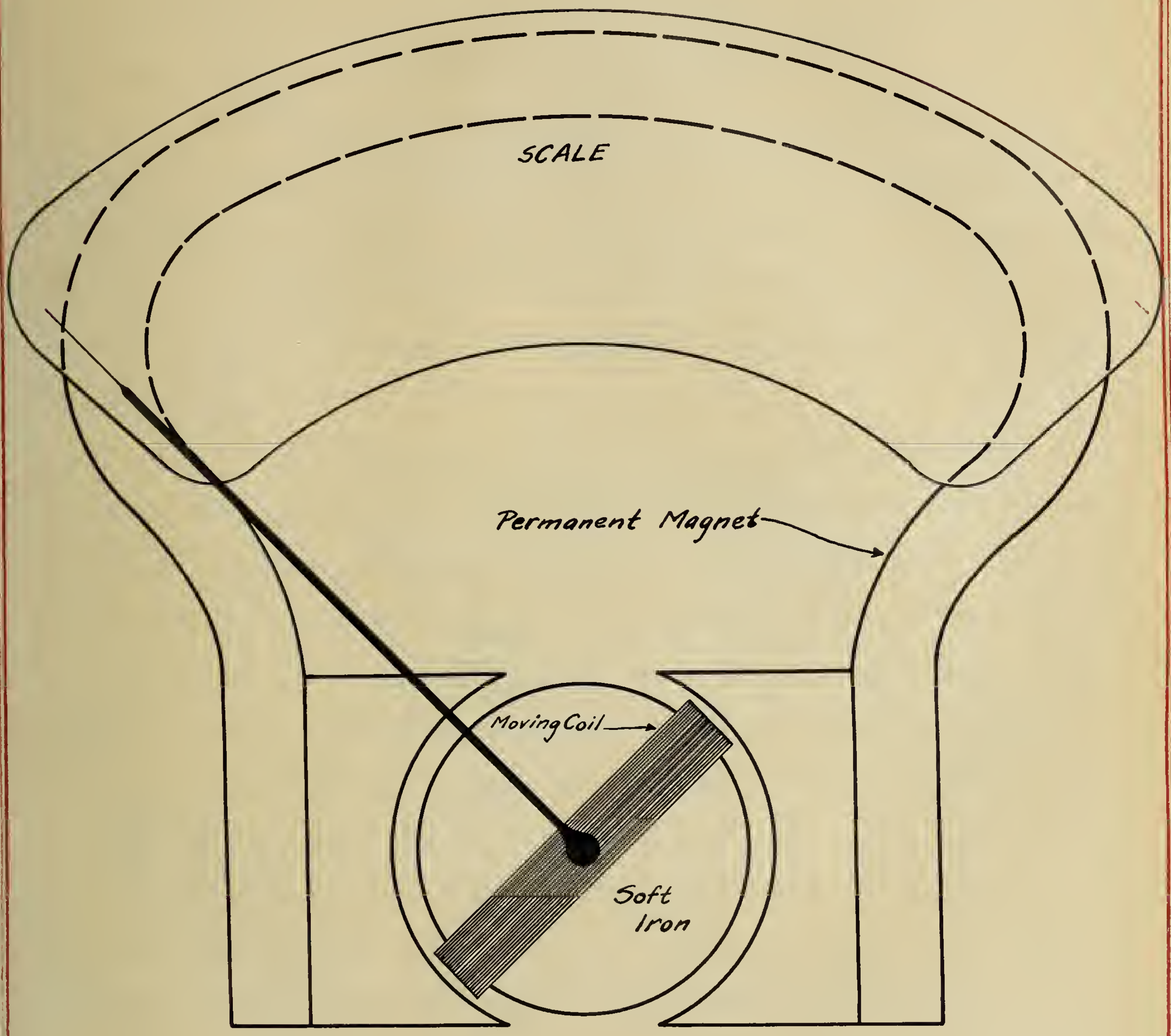



FIG. 7.

send sufficient current thru it to give full scale deflection. Thus, simply, the instrument is a milliammeter or a millivoltmeter. Calibrated and used as a milliammeter this instrument is quite reliable, being practically unaffected by temperature changes. As a voltmeter, however, its indications are very susceptible to temperature changes for the coil is of copper, the resistive temperature coefficient of which is high.

By the introduction of other resistances, this same instrument is readily converted into either a voltmeter or an ammeter of greater capacity. For use as a voltmeter a comparatively high value of resistance is connected in series with the moving coil in order to cut down the current so as to give full scale deflection at the maximum voltage. If this resistance be made of an alloy, such as manganin, having a practically negligible temperature coefficient it is evident, if the added resistance is great as compared with that of the coil, that the combined temperature coefficient is practically negligible, and consequently the deflections will be proportional to the voltage, practically independent of the temperature. In a voltmeter of this type then, but little trouble is experienced from temperature changes when a resistance made of the proper alloy is used.

If it is desired to measure a current greater than the capacity of the moving coil, it is passed thru a low resistance and the instrument shunted across this resistance. If this low value resistance, or shunt, has a negligible

temperature coefficient, then the drop across it is at all times proportional to the current flowing independent of the temperature. Then, if the millivoltmeter connected across this shunt had a negligible temperature coefficient it would at all times give deflections proportional to this drop and hence proportional to the current. But this condition is not realized. The coil is wound with copper wire, and the shunt is usually of manganin or some similar alloy. Thus while the drop across the shunt is practically independent of temperature, the current thru the millivoltmeter does depend to a considerable extent upon the temperature. This may be remedied to some extent by connecting a manganin resistance in series with the coil, thus cutting down the temperature coefficient of the whole; or even by introducing an alloy such as nickel-manganese-copper having a negative temperature coefficient sufficient to compensate for the positive coefficient of the copper. But a limit is imposed upon the introduction of resistance due to the excessive drop across the shunt. The Weston Electrical Instrument Co. employ this method of compensation, by introducing a resistance of negligible temperature coefficient equal to that of the copper coil, thereby reducing the percent error at the cost of efficiency. It is evident that if the shunt and coil were made of the same material having the same temperature coefficients, and were always at the same temperatures, they would divide the current in the same proportion at all loads for all temperatures, and the above objection would be overcome. But in a commercial instrument it is not likely that they will be at the same temperature, and hence this solution fails. With a manganin shunt, a correction for temperature variation may be made as

follows:

Let $R$ = resistance of shunt

$r$ = resistance of instrument

$= r_o (1 + \alpha t)$, where $r_o$ is the resistance of the instrument at the temperature at which it was calibrated, $\alpha$ is the temperature coefficient = percent increase in resistance per degree rise in temperature and $t$ is the degrees rise in temperature above the calibration temperature.

$I$ = total line current.

$i$ = current thru instrument.

Then

$$i = \frac{R}{R + r} I \qquad (1)$$

And at the temperature of calibration,

$$i = \frac{R}{R + r_o} I \qquad (2)$$

At the temperature at which the reading is taken,

$$i = \frac{R}{R + r_o (1 + \alpha t)} I \qquad (3)$$

And the correction factor to be applied to the reading is (2) divided by (3), or

$$K = \frac{(R + r_o) + r_o \alpha t}{(R + r_o)} \qquad (4)$$

For copper $\alpha$ is positive ~~for copper~~ (about .0038), so that

for a rise in temperature K is greater than 1, and for a fall in temperature K is less than 1. It is also evident from (4) that the greater the value of R as compared with $r_o$, the nearer will K be to unity. If (3) be substracted from (2), and the difference divided by (2), the percent error due to temperature changes is obtained in the following form:-

$$\mathcal{E} = \frac{\alpha t}{\frac{R}{r_o} + 1 + \alpha t} \qquad (5)$$

from which it is seen that the smaller R is as compared with $r_o$, the greater the percent error, approaching $\frac{\alpha t}{1 + \alpha t}$ as a maximum limit. As an example let an instrument having the following constants be considered.

$R = .01\ \omega$ (5 amp. shunt)

$r_o = 25\ \omega$

$t_o = 20°\ C$ (Calibration temperature)

$t_1 = 10°\ C$ (Temperature at which used)

$t = t_1 - t_o = -10$

$\alpha = .0038$

$$\mathcal{E} = \frac{.0038 \times (-10)}{.0004 + 1 + [.0038 \times (-10)]} = -3.95\%$$

In practically all cases the term $\frac{R}{r_o}$ (Eq. 5) is so small as compared with the rest of the denominator as to be negligible, and the expression for the error may with sufficient accuracy be written

$$\mathcal{E} = \frac{\alpha t}{1 + \alpha t} \qquad (6)$$

which involves only the temperature coefficient and the change in temperature, Using (6) in the above example,

$$= - 3.95\% +$$

If a manganin resistance equal to four times the copper resistance, $r_o$, be connected in the millivoltmeter circuit, then the temperature coefficient of this circuit is

$$\alpha_1 = \frac{\alpha}{5} = \frac{.0038}{5} = .00076$$

and

$$\mathcal{E} = \frac{-.0076}{1-.0076} = - .766\%$$

From the above it is evident that the percent error due to temperature changes in an ammeter employing a shunt has a maximum limit of about .4% per degree C, and may be reduced by the introduction of a manganin resistance in series with the coil,-the percent error varying practically as the ratio of the copper resistance to the total resistance in the millivoltmeter circuit.

Another very ingenius method for temperature compensation in an instrument of this type is due to Mr. A. Campbell, and the bridge arrangement of the circuits in illustrated in Fig. 8. The various arms are of copper and manganin as indicated and their resistances bear the 3 to 1 ratios indicated. The theory of this arrangement, showing that by neglecting the second power of the temperature coefficient perfect compensation is obtained, follows. The

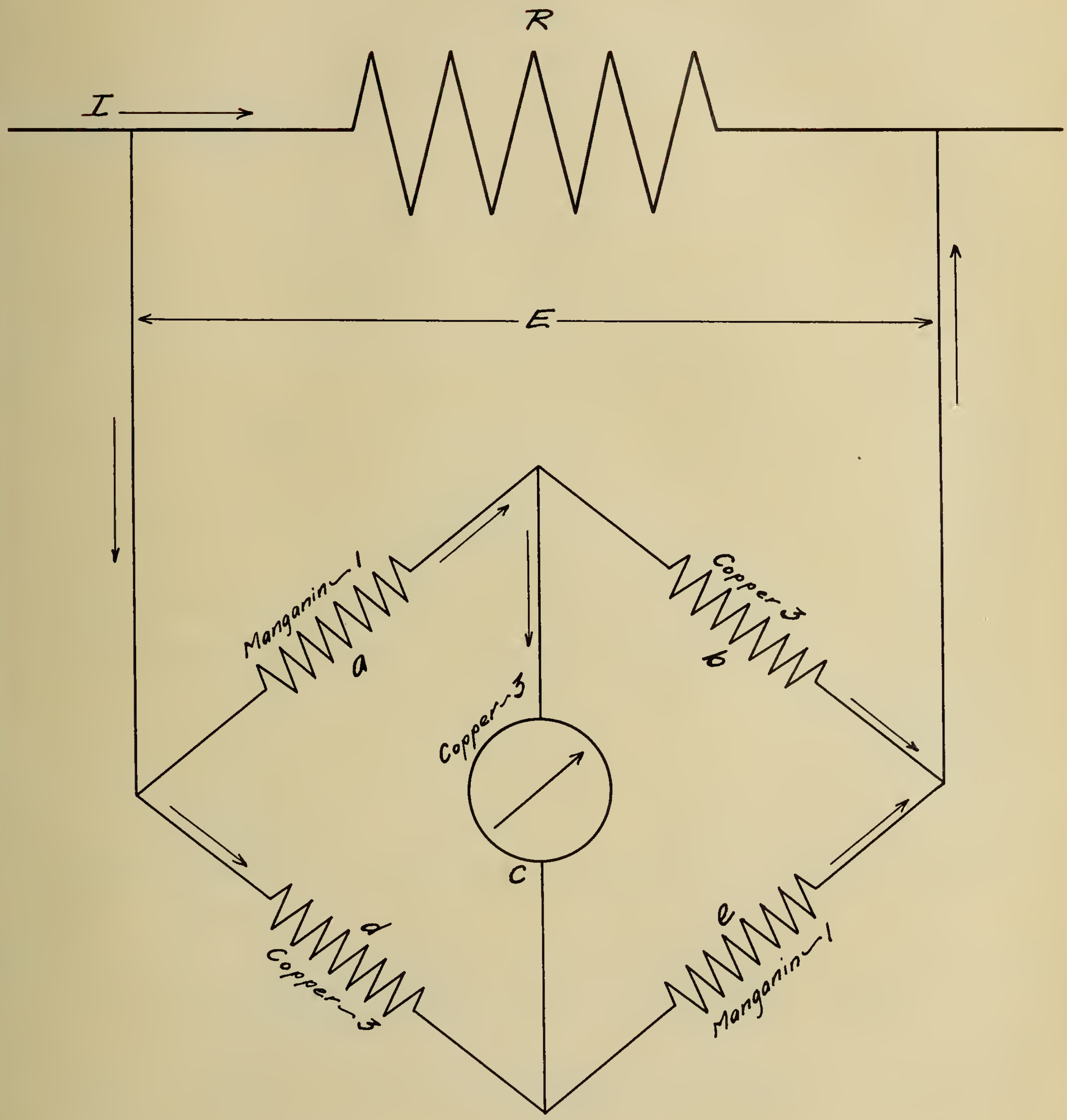



FIG. 8.

The lower case subscripts refer to the different arms indicated in Fig. 8, and e is the voltage drop across each, R the resistance of each and i the current in each. E is the voltage across the shunt.

$$e_a = R_a\, i_a \tag{7}$$

$$e_b = R_b\,(1 + \alpha t)\, i_b \tag{8}$$

$$e_c = R_c\,(1 + \alpha t)\, i_c \tag{9}$$

$$e_d = e_b \qquad \text{and} \qquad e_c = e_a$$

$$e_d = e_a + e_c = e_b \tag{10}$$

$$e_a + e_b = E \tag{11}$$

$$i_a = i_b + i_c \tag{12}$$

Substituting equations (7), (8) and (9) in (10) and (11), multiplying (12) by $R_a$, and transposing,

$$\begin{aligned} R_a\, i_a - R_b\,(1 + \alpha t)\, i_b + R_c\,(1 + \alpha t)\, i_c \quad &= 0 \\ R_a\, i_a + R_b\,(1 + \alpha t)\, i_b \qquad - E &= 0 \\ R_a\, i_a - \quad R_a\, i_b \quad - \quad R_a\, i_c \quad &= 0 \end{aligned}$$

Solving these three simulataneous equations for $i_c$, gives

$$i_c = \frac{\left[R_b\,(1 + \alpha t) - R_a\right]}{\left[(2\, R_a R_b + R_a R_c)(1 + \alpha t) + R_b R_c\,(1 + \alpha t)^2\right]}\, E \tag{13}$$

Substituting in equation (13), $R_a = 1$, $R_b = R_c = 3$, and reducing,

$$i_c = \frac{1}{9}\,\frac{(2 + 3\ \alpha t)}{(2 + 3\quad t + \alpha^2 t^2)}\, E \tag{14}$$

which, if the term $\alpha^2 t^2$ be neglected, reduces to

$$i_c = \frac{1}{9}\, E \tag{15}$$

or the current thru the coil is directly proportional to the voltage across the shunt. If, as before, $\alpha$ = .0038 and t = - 1C, and these values be substituted in (14)

$$i_c = \frac{1}{9} \frac{2 + [3 \times .0038 \times (-10)]}{2 + [3 \times .0038 \times (-10)] + (.0038)^2} E$$

$$= \frac{1}{9} \frac{1.886}{1.88744} E = \frac{1}{9} E -$$

which shows that the omission of the term containing the square of the temperature coefficient is warranted, and that almost perfect compensation is obtained for ordinary changes in temperature.

Another source of occassional trouble in this type of ammeter ariees from thermal E. M. F.s generated at the contact between the copper and manganin of the shunt. This may be overcome by connecting the millivoltmeter leads directly to the manganin; or by constructing a shunt such as shown in Fig. 9, where the millivoltmeter current is brought back thru the additional manganin strip M, so that both millivoltmeter leads are connected to the same end of the shunt. In this way the thermal E. M. F.s are made to neutralize each other.

Before leaving the type of instrument a few words in a general way may be given. Referring to Fig. 7 it will be seen that there is a core of soft iron between the pole pieces about which the coil rotates. This core serves the purpose not only of providing an easy passage for the flux from one pole to the other thus furnishing large torque, but

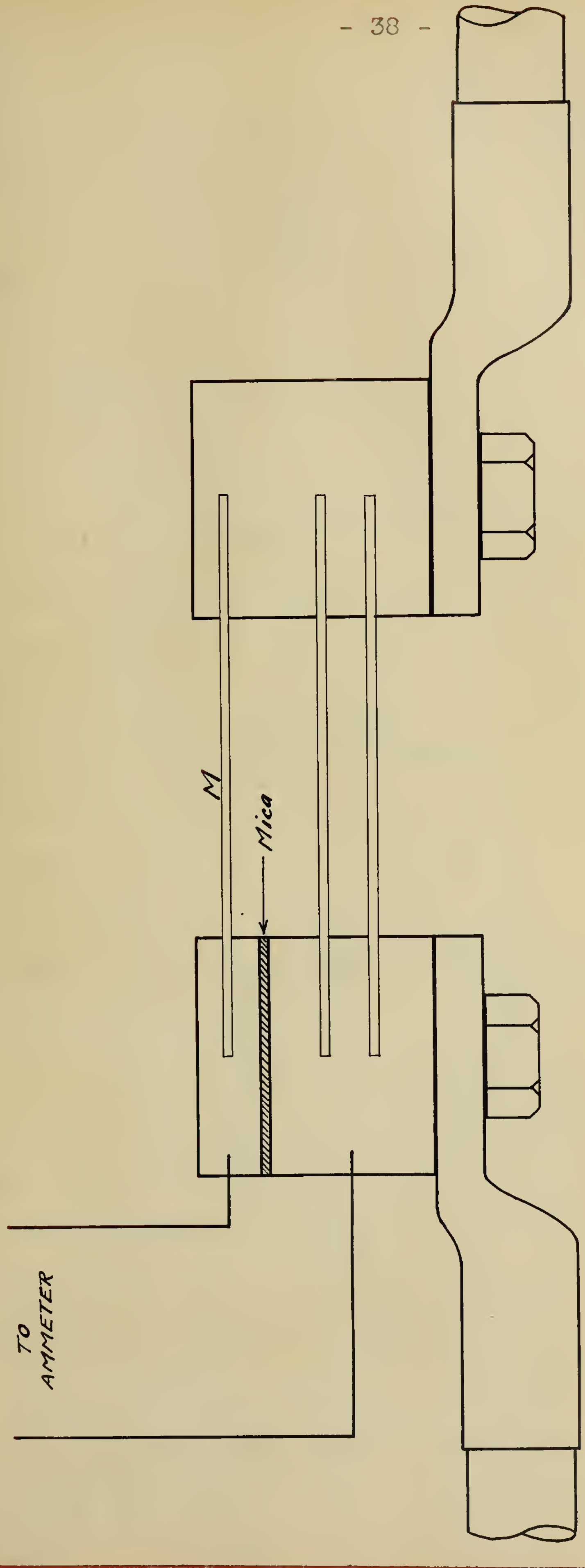



FIG. 9

also of so distributing the flux that the flux threading the coil is practically a constant value for all positions of the coil. This means that the torque is practically proportional to the current in the coil for all positions of the coil, and since the resisting torque is directly proportional to the deflection, equal scale divisions from no load to full load are obtained. As has been pointed out, high torque for comparatively small values of current, and small inertia of the moving parts are obtained in such an instrument; but the permanency of calibration depends upon the permanency of the permanent magnets, and therefore it is of the utmost importance that these be properly treated and "aged" before final calibration. The "aging" of the magnets is done artificially and in a comparatively short time, but although very interesting hardly falls within the scope of this paper. On the whole this type of commercial instrument seems to be the most satisfactory, reliable and efficient that has so far been devised for the measurement of D. C. current and voltage.

Instruments in most common use that will operate on A. C. as well as D. C. fall under Cases 2, 4 and 5. Cases 2 and 4 are applicable to ammeters as well as voltmeters, but the inadequacy of shunts in A. C. meters introduces the mechanical difficulty of a heavy moving element, and thus limits case 5 to voltmeters and wattmeters.

The Weston Electrical Instrument Co. manufacture a type of instrument that comes under Case 2, and which is shown diagramatically in Fig. 10. Although this instrument

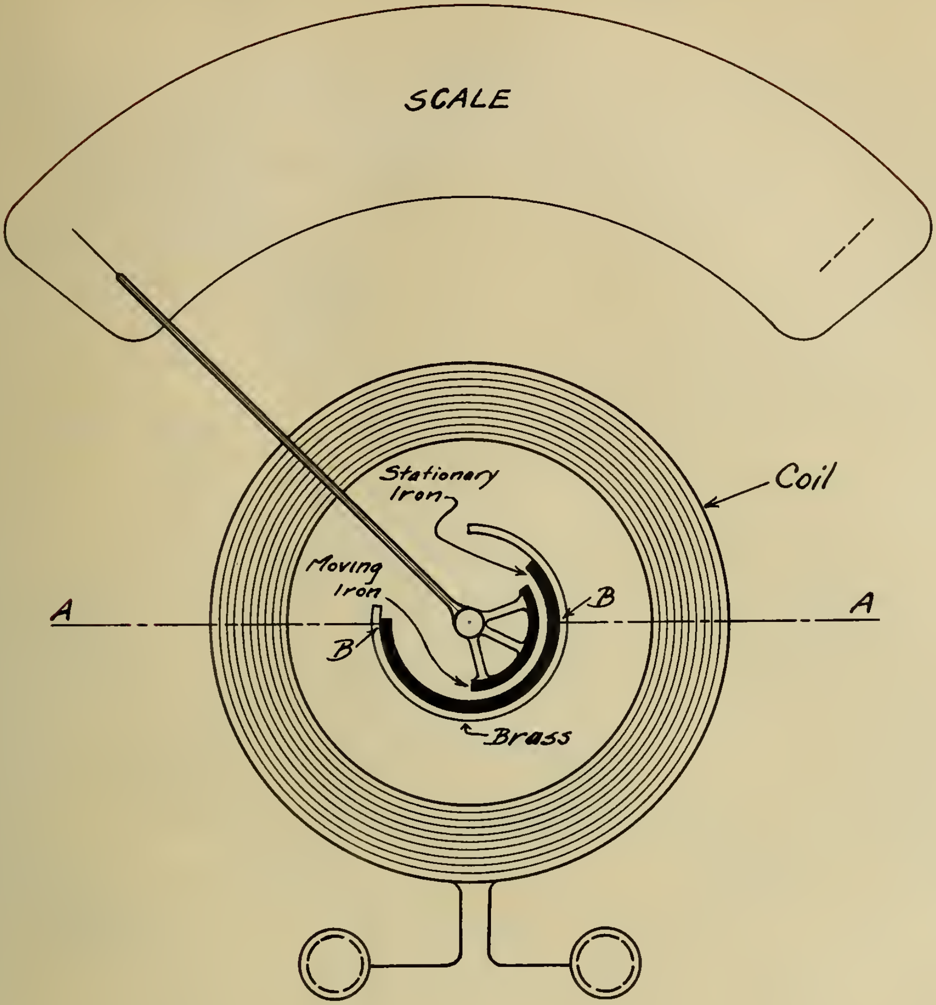
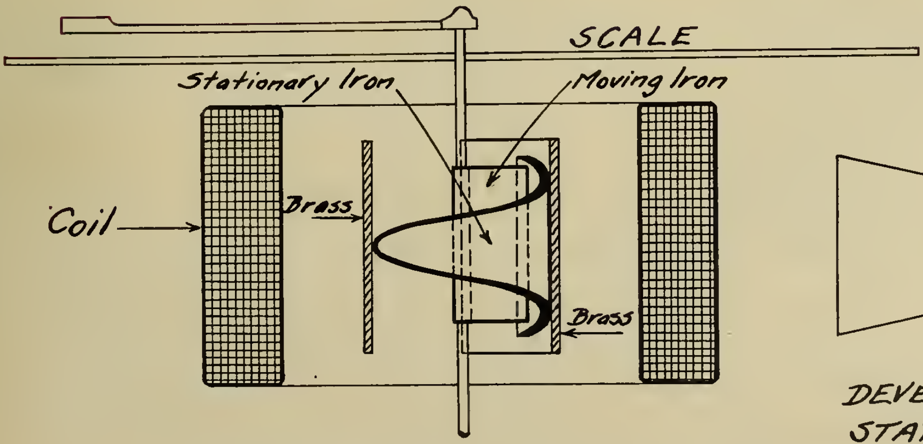
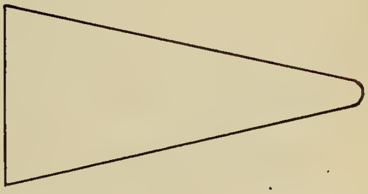



FIG. 10




SECTION THRU A-B, B-A



DEVELOPEMENT OF STATIONARY IRON.

will give readings on D. C., it is for reasons explained in Chapter I essentially an A. C. instrument. The current is passed thru a fixed coil the plane of which is parallel to the scale, and which may be wound either with a few turns of heavy wire, or many turns of five wire, thus adapting the instrument for use as an ammeter or a voltmeter. Inside this coil and coaxial with it is a fixed cylindrical element not quite closed on itself, and made up of two metals, brass and soft iron. The iron is the important part, the brass acting merely as a support for it. The iron is a sheet cut in the form of a long wedge and wrapped around the cylinder, as indicated. The moving element which is just inside the stationary one and arranged as shown in the figure is a segment of a cylinder whose altitude is equal to the base of the wedge, and is made entirely of soft iron. In the zero position this cylindrical moving element is opposite the base of the wedge. Now when a current flows in the coil, at any instant, the adjacent ends of the two iron elements will be at the same polarity, and the force exerted between them will be one of repulsion. The result will be that the pivoted element will move down along the wedge, thus deflecting the pointer, until the repulsive force is held in equilibrium by the spiral spring attached to the shaft. This type of instrument is practically unaffected by frequency, but slightly by ordinary variations in wave form, and is quite reliable so long as the molecular structure of the iron remains unchanged.

Case 4 similarly to Case 2 may be applied to ammeters as well as voltmeters, and is, for similar reasons, essentially applicable to A. C. meters. The usual form in which this principle is employed is that of a coil at the center of which is a small iron vane or vanes fastened to a shaft carrying a pointer, and set at an angle to the lines of force thru the coil. When a current is passed thru the coil, these vanes tend to set themselves parallel to the lines of force and in so doing produce a torque which rotates the shaft until equilibrium is reached against a spring. The coil is stationary, and may obviously be of many turns of fine wire, or of a few turns of heavy wire, thus adapting this arrangement for an ammeter as well as a voltmeter.

It is upon this principle that perhaps most of the A. C. ammeters at the present time are constructed. The Hoyts A. C. Ammeter is sketched diagramatically in Fig. 11. In this instrument the the vane is kept in the position shown by means of a torsion head, and this position is indicated by a pointer at (a) attached to the vane. To the torsion head is attached a pointer (b) which gives indications on a circular scale. When (a) has been brought back to its zero position the indication of (b) gives the value of current flowing. As may be seen, there are two coils, one a 10 ampere coil and the other a one ampere coil, so that the instrument may be used for two ranges of current. The scale divisions are small for low values of current, but become very large for higher values. The reason for this is obvious when

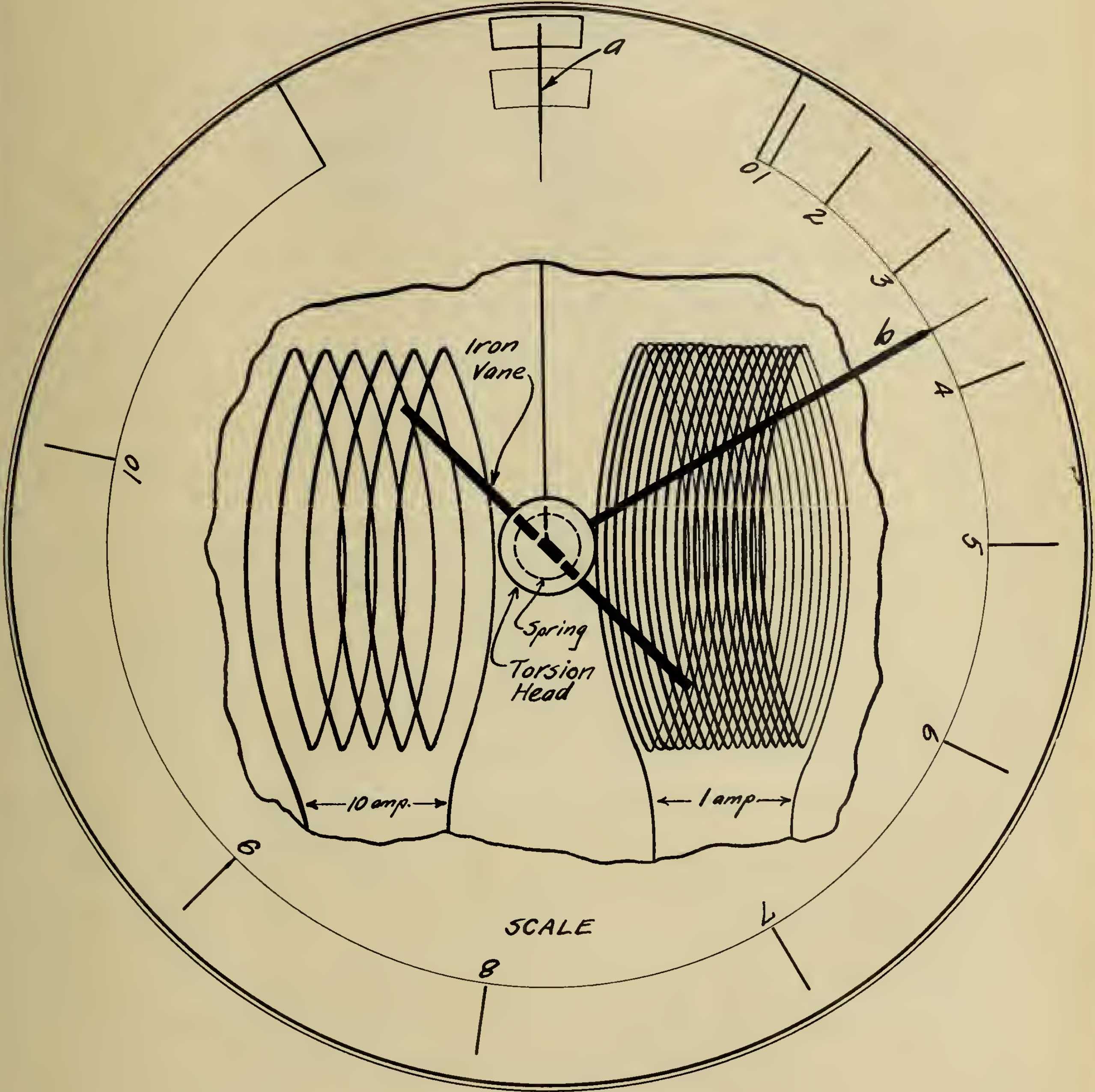

a
b
0
1
2
3
4
5
6
7
8
9
10
Iron Vane
Spring
Torsion Head
10 amp.
1 amp
SCALE

FIG. 11.

it is remembered that the torque varies as the square of the current except for the deviation caused by the variation of permeability with current.

The General Electric Co.'s ammeters of the $P_3$ and $P_4$ type also operate on this principle. A skeleton sketch of these is given in Fig. 12. These instruments are direct reading, i.e., they give the value of current by direct deflection. The stationary coil is inclined at an angle of 45° to the plane of the scale, as shown. The shaft is at right angles to the scale, and the pointer is of course at right angles to the shaft. Three parallel soft iron vanes are fastened to the shaft at the center of the coil in such a position that their planes are at 45° to both the shaft and the pointer. To protect the field of the instrument from external fields the coils and armature are enclosed in a laminated iron case. In the $P_4$ instruments the coil is divided into two sections, which sections may be connected either in series or in parallel, thus giving the instrument two ranges, one twice as great as the other. The switch that accomplishes this is in one corner of the box, and is so arranged that the reading may be switched from one scale to the other without breaking the circuit. A current flowing in the coil sets up a field at right angles to the plane of the coil, and the iron vanes in tending to set themselves parallel to this field produce a torque which rotates the shaft against the action of a spiral spring. The scale divisions are small at the beginning and end, and largest

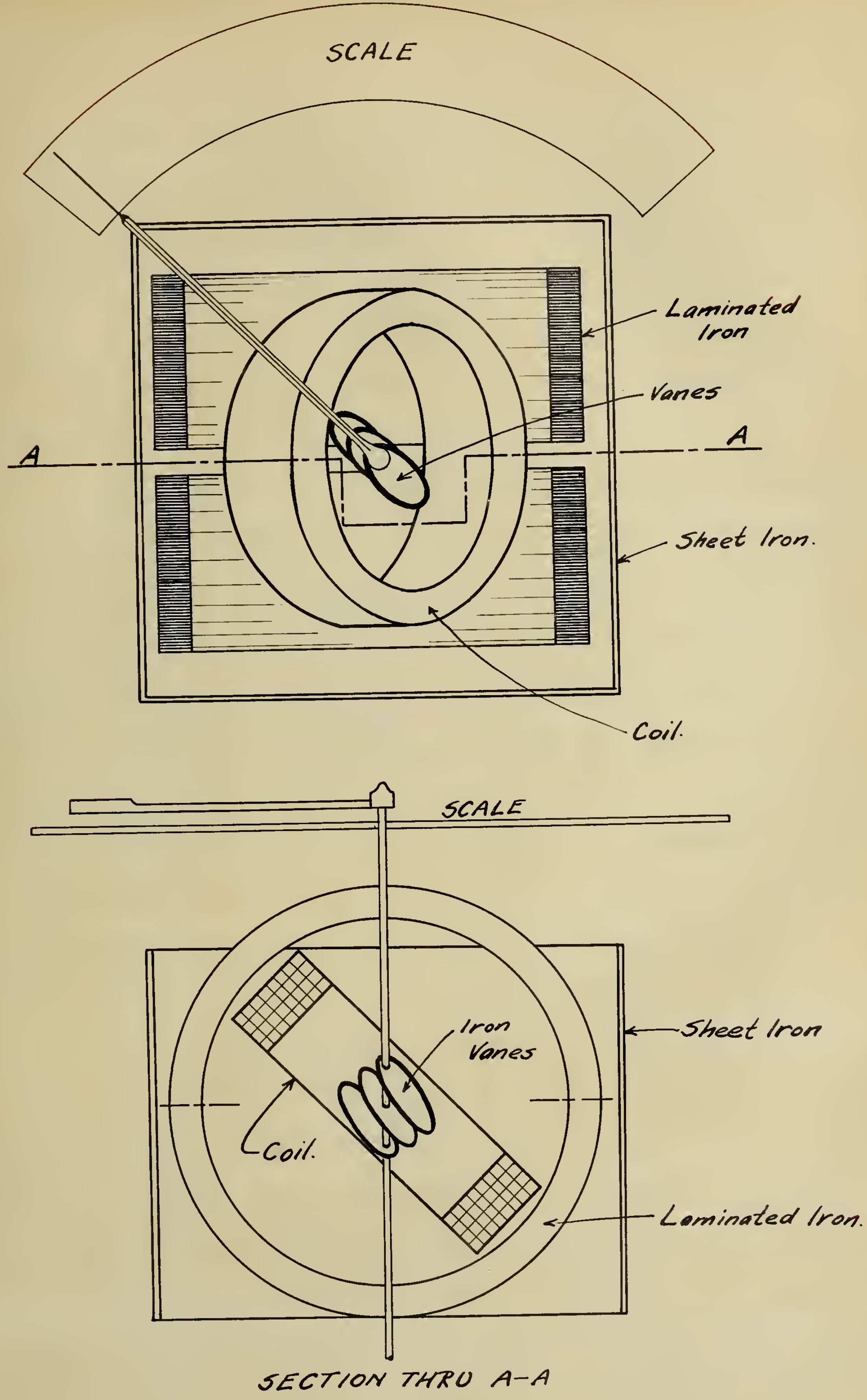

SCALE
Laminated Iron
Vanes
A
A
Sheet Iron.
Coil.
SCALE
Iron Vanes
Sheet Iron
Coil.
Laminated Iron.
SECTION THRU A-A

FIG. 12.

near the middle. For low current values the torque is small because the force exerted is small, and near the end of the scale the rate of increase of torque decreases due to decreasing moment arm. Like the preceding instruments, this type is practically unaffected by frequency as an ammeter, and is quite reliable so long as the molecular structure of the iron remains unchanged.

Under Case 5 comes the Siemens dynamometer type of instrument in which the torsion head is omitted, and for the fibre suspension is substituted jewel bearings, the instrument being made direct reading by attaching a pointer to the moving coil. An instrument of this type may be used equally as well on D. C. as on A. C. with the exception that external fields give more serious trouble when using D. C. than when using A. C. As a commercial instrument for D. C. measurements, however, one of the D'Arsonval type is to be preferred over one of this type for there a strong field is supplied by permanent magnets, whereas in this type of meter the strong field must be set up by the current itself. This means many ampere turns, and an unavoidable increase in current which is undesirable. But in the case of A. C. permanent magnets cannot be resorted to for the production of an unidirectional average value of torque; and the torque must be established entirely by the current flowing thru the instrument. If it were possible, practically, to make the moving element of any size wire desired, then such a dynamometer type instrument with the moving and

stationary coils connected in series could be used for either an ammeter or a voltmeter. But a limit far below the capacity required for an ammeter is placed upon the size of the moving coil, and since a shunt is not practical for A. C., this type of instrument is not available for an ammeter. It is as an A. C. voltmeter and wattmeter, which will be considered later, that it finds its widest application. With the moving and stationary coils of fine wire connected in series and A. C. voltage impressed across them, the instrument will read truely the r.m.s. of the current flowing thru it, as is evident from a consideration of the fact that the instantaneous value of torque is directly proportional to the square of the instantaneous value of current. If now, the coils were only resistance, the instrument would also read truly the r.m.s. of the impressed voltage. But the coils are reactive to some extent and, therefore, the current flowing depends upon the frequency, and consequently upon the wave shape. As ordinarily constructed, however, such that the resistance is very high as compared with the reactance, the effects of variations in frequency and wave shape within commercial limits is not serious, except, perhaps, in the lower range voltmeters. Above 200 cycles the error generally becomes quite appreciable. The above discussion involving the effect of the reactance of an A. C. voltmeter on its sensitiveness to frequency changes and wave form, applies equally well to all the A. C. voltmeters considered.

Fig. 13 is a sketch of the arrangement of the coils

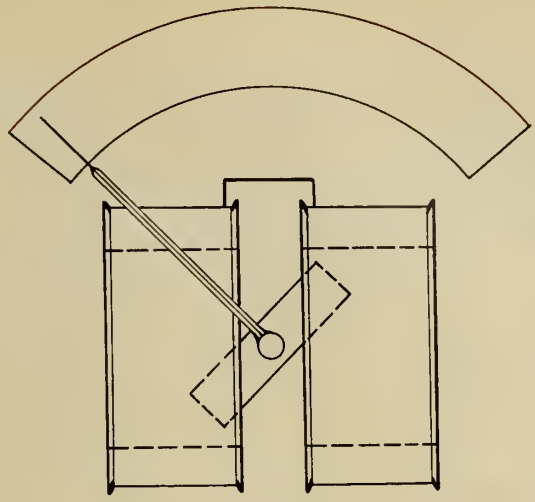


*FIG. 13*

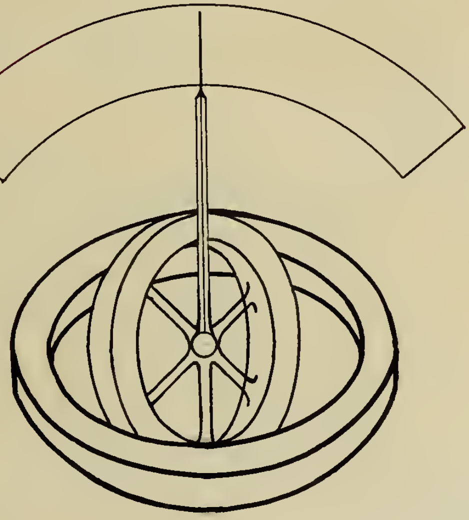


*FIG. 14*

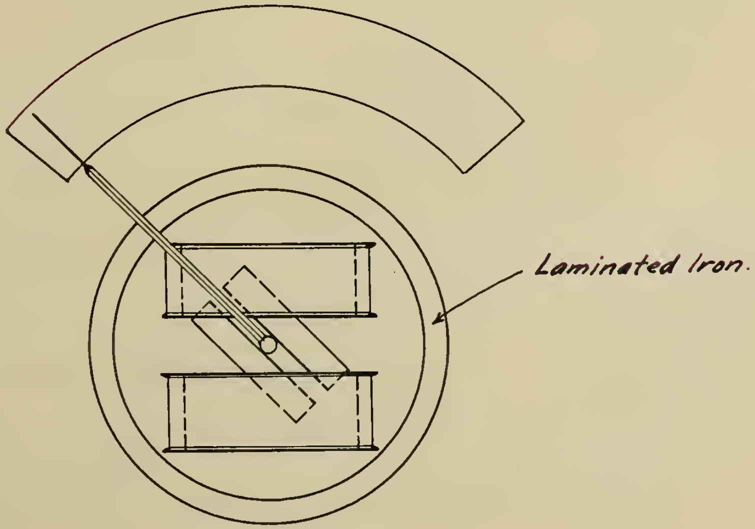



*FIG. 15*

in a Weston voltmeter, and Fig. 14 of the arrangement of the coils in a Thomson voltmeter of this type. The stationary coil is inclined 45° to the plane of the scale, and the moving coil is inclined 45° to the plane of the shaft and pointer. With this arrangement the maximum scale devisions come at about 100 volts on a 130 volt scale, which is very satisfactory. In this particular instrument the current is lead into the moving coil thru one spiral spring, out thru another on the same end of the shaft, and still a third spiral spring on the same end of the shaft serves for adjustment.

Fig. 15 is a diagram of the arrangement of the coils in a General Electric Co. Type $P_3$ voltmeter. The moving coil swings thru 90° and is 45° to the fixed coil at either end of the scale. The maximum scale divisions occur at about 180 volts on a 300 volt scale. In this instrument one side of the moving coil is grounded, and the field of the instrument is protected from stray fields by a laminated iron shield which completely surrounds the coils.

Induction Meters, coming under Case 8, that will operate on A. C. only will be considered briefly. It is known that if a conductor be placed in a varying magnetic field there is induced in it an electromotive force which sets up a flow of current in the conductor, and that this current sets up a field which reacts on this inducing field thus producing a force between the conductor carrying the

primary current and the conductor carrying the induced current. This principle has been employed in the construction of meters. Fig. 16 is a diagrammatic sketch of a Westinghouse Portable Ammeter using this principle. It consists of a magnetic circuit of laminated iron with two poles as shown between which is an aluminum cylinder fastened to a shaft carrying a pointer and turning against the action of a spring. On this iron core are two windings, a primary winding and a secondary winding. The primary windings are the largest, and are wound on the legs of the iron circuit marked AA. The terminals of the coil on each leg are brought out to binding posts so that these primary coils may be connected either in parallel or in series in order to obtain two ranges. On these same legs are also wound the secondary coils, connected in series, and connected across the terminals of the two coils B B, wound around the poles as shown, and which coils are connected in series thru a resistance. This arrangement gives two fluxes displaced in time by an angle depending upon the resistance and reactance of the secondary circuit, and these two fluxes so displaced result in a rotary field between the poles, which by the principle of induction produces a torque tending to rotate the aluminium cylinder in a definite direction. The pointer attached to the shaft is deflected until equilibrium is reached and readings are taken from a complete circular scale. The diagram of connections is also given in Fig. 16.

The chief advantage of this instrument seems to be

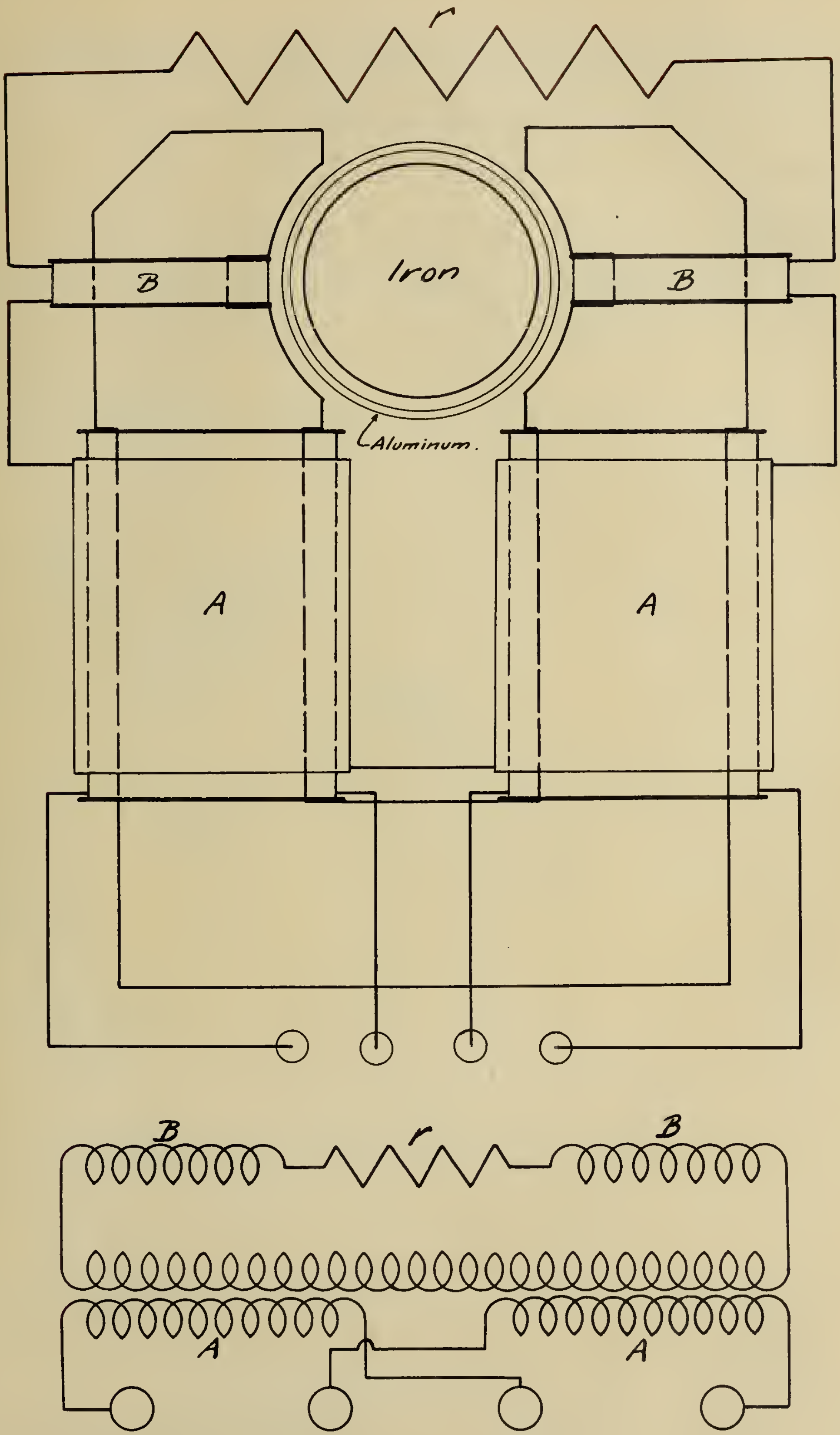



FIG. 16

the large scale that the circular arrangement makes possible. To off set this it has the disadvantages of containing[a] comparatively large amount of iron which makes it noticeably heavy; and, more serious than this, of being comparatively sensitive to frequency and consequently wave-form, as shown by the fact that there is almost 2% difference in the readings for 25 and 60 cycle.

# CHAPTER III

## THE INDICATING WATTMETER

The indicating wattmeter again brings forward the Siemens dynamometer type of instrument. Such wattmeters are similar in construction to the voltmeters of this type previously described, but the stationary coil instead of consisting of many turns of fine wire is wound with a few turns of heavy wire and is connected in series with the load, while the moving coil wound as in a voltmeter is connected, in series with a resistance, across the line. If for a given position of the pointer there is a definite average value of the instantaneous products of current and voltage in the circuit in which the instrument is connected, then the instrument may be used to measure power. That this condition is fulfilled if the moving coil be considered as only a resistance, is evident from a consideration of the expression

$$t = K\, i_1\, i_2$$

where $t$ is the instantaneous value of torque, and $i_1$ and $i_2$ are the instantaneous values of current in the stationary and moving coils respectively. Now $i_1$ is one of the factors of direct interest, and may therefore be anything at all. But $i_2$ is of importance only in its relation to the voltage impressed upon the moving coil circuit. If this circuit be only a resistance not subject to the influence of other

fields such as that of the stationary coil, then, at each instant the current in the moving coil is proportional to the impressed voltage, $e_2$; and the above expression for torque may be written,

$$t = K' i_1 e_2$$

whence the average value of torque is proportional to the average value of power, and therefore a given position of the pointer represents a definite value of power. This is the ideal condition; but two things have been assumed to reach this end, first, that the moving coil is without self-induction and, second, that there is no mutual inductance between it and the stationary coil. In the Siemens electric dynamometer where a torsion head is used to keep the coils always at right angles the second of these assumptions is practically obtained, but in a deflecting instrument this assumption does not hold. The first assumption is, of course, impossible in either instrument. Usually, the resistance of the moving coil or potential circuit is so very high as compared with its inductance at ordinary frequencies, that the inductance is generally neglected. It is evident, however, that if these be not negligible, the current in the potential circuit will not at every instant be proportional to the impressed voltage, and consequently the reading of the instrument will be in error, which error will not be constant for a given power but will increase very rapidly at low power factors. If a sine wave of e.m.f. and

current be assumed this increase in the error due to the inductance of the potential circuit may be shown by a vector diagram as in Fig. 17, where the lag angle $\theta$ in the potential circuit is somewhat exaggerated for the sake of clearness. In this diagram the vector $I_s$ represents the constant current in the stationary coil. The line O P represents the constant true power in the circuit for the two power factors to be considered. In the first case the voltage impressed upon the potential circuit is E', the power factor is cos $\alpha$ and the current in the potential circuit is $I_m'$ lagging behind E' an angle $\theta$ depending upon the inductance and resistance of the potential circuit. The reading of the wattmeter in this case is represented by the line O A (as will be demonstrated later), and A P represents the error. If now the power factor be considered as decreasing until it becomes equal to cos $\beta$ , and the impressed voltage as increasing until it reaches the valeu E" such that the true power in the circuit remains constant, then since the angle $\theta$ and the proportionallity between E and $I_m$ has remained constant, provided of course the frequency has not changed, $I_m$ will be as represented by $I_m''$, and the reading of the instrument will be represented by the line O B. In this case the error is represented by the line B P which obviously is greater than A P. The above, of course, is for simple harmonic variations of current and voltage at a constant frequency. The inductance of the potential circuit further complicates matters by introducing a variation in indication

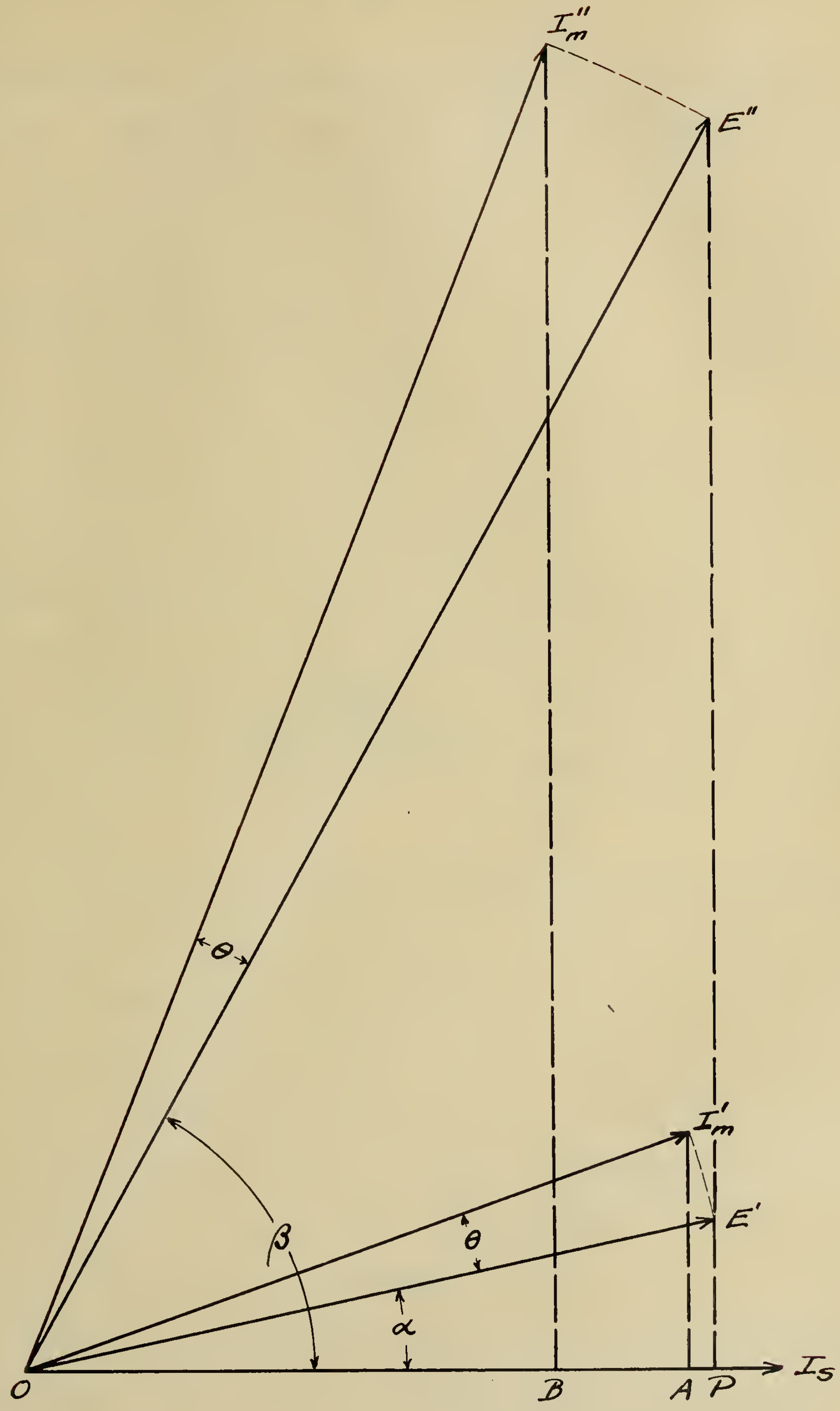
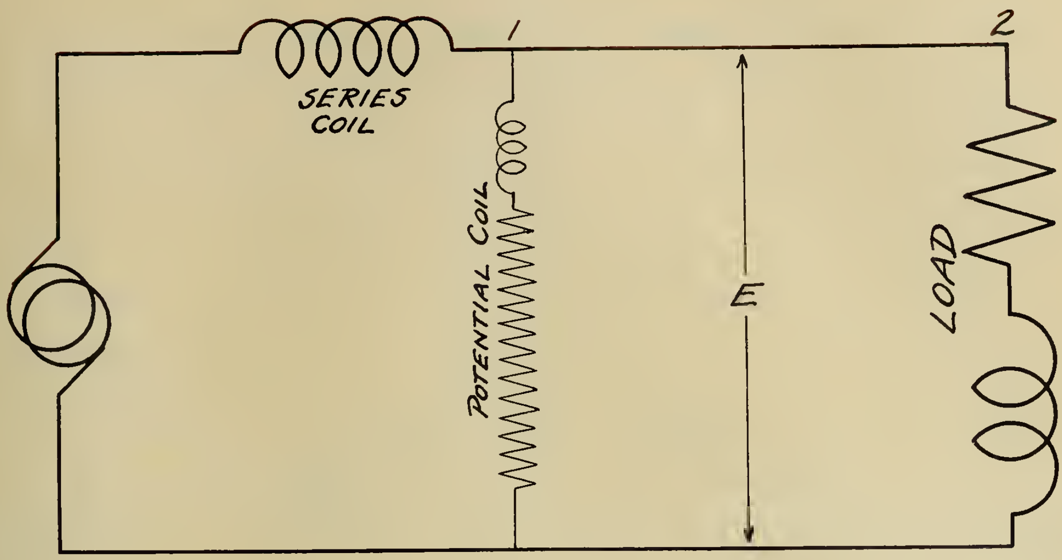


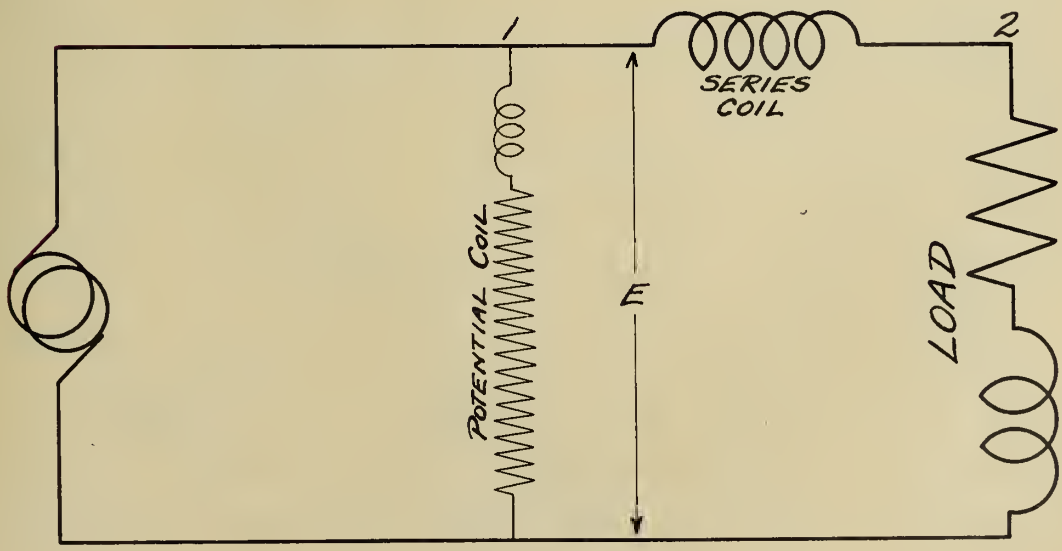

FIG. 17

with the frequency and consequently with the wave shape of the voltage. There are, of course, other sources of error common to all instruments, such as friction, warping, distortion or disintegration of some vital part, temperature etc. As regards temperature, it may again be said that the high resistance of negligible temperature coefficient connected in series with the moving coil reduces to a minimum the effects of temperature changes upon this instrument. In what follows, therefore, these sources of error will be neglected, and attention directed to those already presented, and which are peculiar to the wattmeter. But before beginning the discussion, consider the ways in which a wattmeter may be connected to measure the power consumed by a load.

The potential circuit may be connected on the same side of the series coil as the load, as shown in Fig. 18 (a), in which case both the load current and potential coil current flow thru the series coil. With an ideal instrument, in this case, the average product of the sum of the currents thru circuits 1 and 2 and the voltage E impressed upon them is measured, or, in other words, the wattmeter measures the power consumed in its own potential circuit as well as in the load. Again, the series coil of the wattmeter may be connected inside the potential circuit, as shown in Fig. 18 (b), in which case the ideal instrument measures the average product of current thru the series coil and load, and voltage, E, impressed across the two in series, or, in other


SERIES COIL
1
2
POTENTIAL COIL
E
LOAD

*a*


1
2
SERIES COIL
POTENTIAL COIL
E
LOAD

*b*

FIG. 18.

words, the instrument measures the power consumed in its own series coil as well as in the load. In either case, then, with an ideal instrument it is necessary to make a correction to the reading to obtain the true power of the load alone; in the first case by subtracting $\frac{E^2}{r_1}$ from the reading where $r_1$ is the resistance of the potential circuit, and in the second case by subtracting $I^2 r_s$ where I is the load current and $r_s$ is the resistance of the series coil. Generally, in commercial measurements, these instruments losses are so small as compared with the load that these corrections are not made, but in dealing with small power measurements their magnitude becomes quite appreciable in proportion, and they must be corrected for.

A word here as to which connection is most suitable for a given condition will not be amiss. In general, it may be said that the first connection with the potential coil inside the series coil is the best for loads in which the voltage is low and the current high, while the second connection may be used where the current is low and the voltage high. The chief point in this matter is to use the connection which gives the lowest instrument loss entering the reading. For high current and low voltage the potential coil loss is likely to be less than the series coil loss and hence the reason for the first connection in this case; while with high voltage and low current the series coil loss may be less than the potential coil loss, and hence the reason for the second connection in this case.

With the first connection it is evident that with no load connected the meter gives a reading which measures the loss in the potential circuit. If, now, another coil of fine wire of the same number of turns as the series coil be wound over the series coil, and this coil connected in series with the potential coil so that when a current flows thru the circuit with no load connected, the ampere turns of this compensating coil will just neutralize those of the series coil, then the current of this potential coil flowing thru the series coil will produce no torque, and the power consumed by the potential circuit will thus be automatically eliminated from thereadings, so that the instrument indications give the power consumed in the load only. Such an arrangement is shown diagramatically in Fig. 19.

A consideration of the magnitude and corrections of characteristic wattmeter errors follows. To begin with the voltages and currents in the systems considered will be assumed to be sinple harmonic variations. This enables them to be treated by the complex quantity method which for convenience and clearness will be found very expedient.

Let it be assumed that a voltage, $e = E \sin \theta$, is impressed upon a reactive load thus causing a current $i = I \sin (\theta + \alpha)$ to flow. The instantaneous value of power consumed by the load is $e\ i$, and the average value of power consumed under uniform condition is the average value for a cycle, or

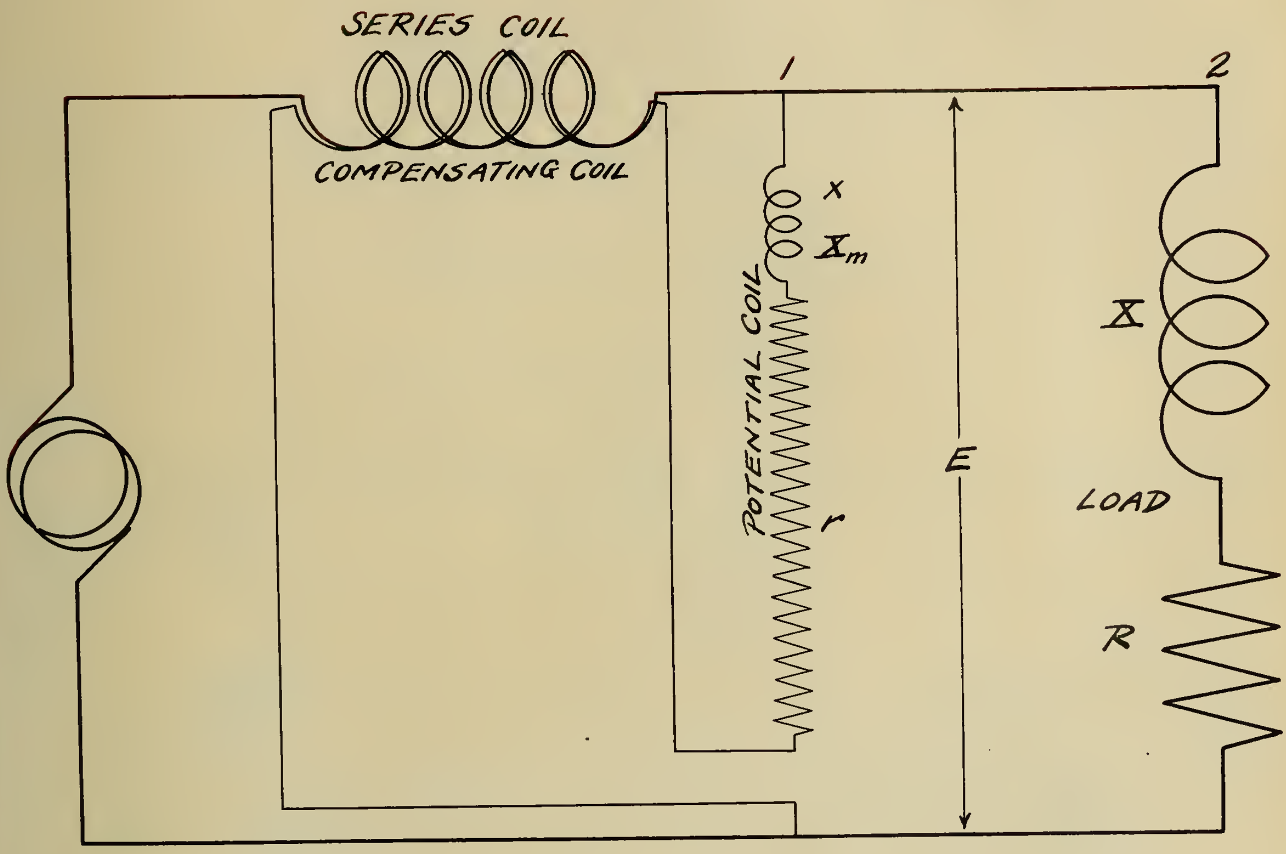

SERIES COIL
COMPENSATING COIL
1
2
x
$X_m$
POTENTIAL COIL
r
E
X
LOAD
R

FIG. 19

$$P = \frac{1}{2\,\pi} \int_{o}^{2\,\pi} e\ i\ d\ \theta$$

$$= \frac{E\ I}{2\ \pi} \int_{o}^{2\ \pi} \sin\theta \sin\ (\theta + \alpha\ )\ d\ \theta$$

The evolution of this integral gives

$$P = \frac{E\ I}{2} \cos\alpha = \frac{E}{\sqrt{2}} \times \frac{I}{\sqrt{2}} \cos\alpha$$

$$= E_{eff.} \times I_{eff.} \times \cos\alpha$$

In the case of the wattmeter the instantaneous value of torque is

$$t = K\ i_1\ i_2$$

If the phase difference between the two currents is the angle $\beta$ , and $I_1$ and $I_2$ are their effective values respectively, then the average value of torque, obtained similarly to the above, is

$$T = K\ I_1\ I_2 \cos\beta$$

and it is evident that a vector diagram of the operation of the wattmeter may at once be drawn. Let the compensated wattmeter, for which the diagram is given in Fig. 19, be the one considered. In this case the current of the load may be considered as the current in the series coil. Fig. 20 gives the vector diagram for such a meter where the current in the series coil, or load current, $I_s$ is the reference vector.

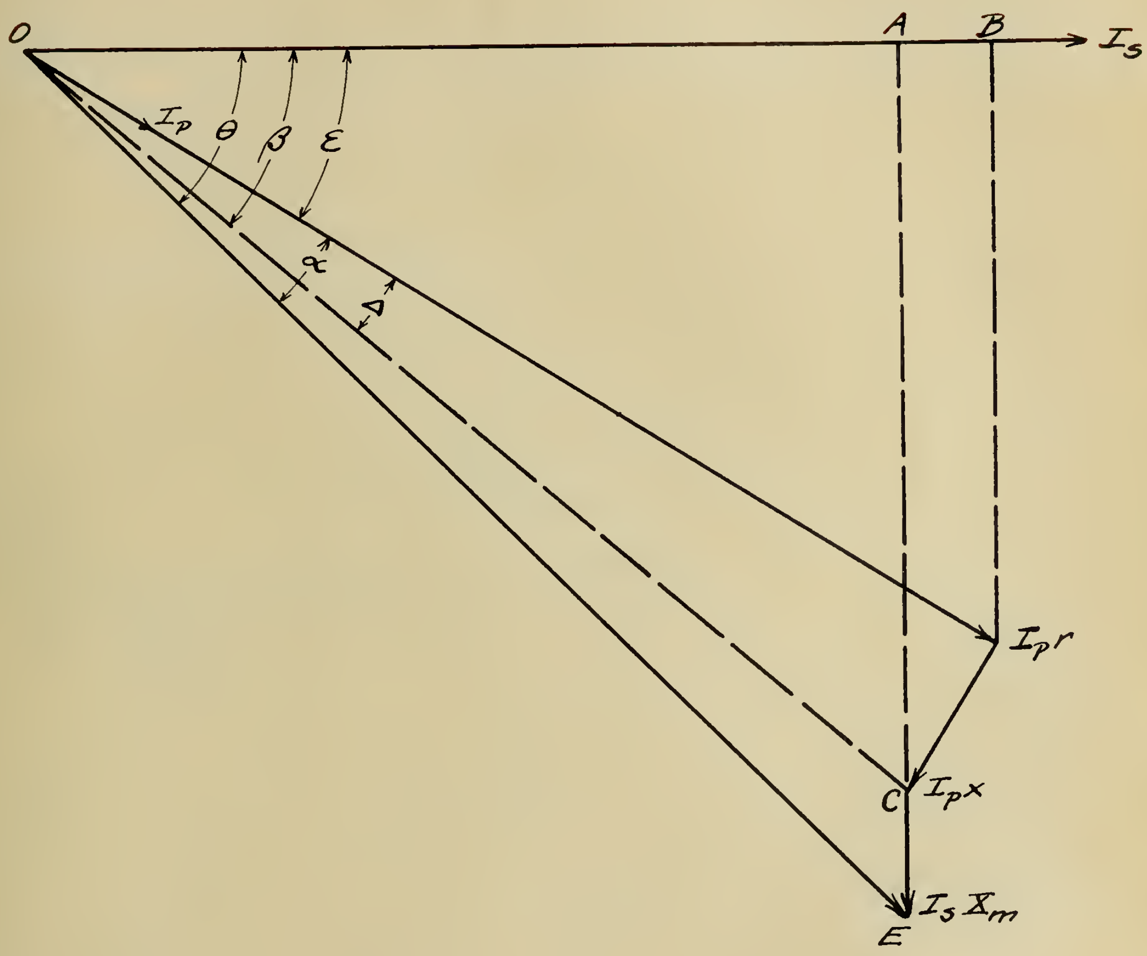



FIG. 20.

If the load be assumed to be inductive, the voltage across the load and potential coil will lead this current by an angle $\theta$ ($\theta = \text{arc tan } \frac{X}{R}$), where X and R are the inductance and resistance respectively of the load. The true power is then represented by the line O A. The current $I_p$ in the potential circuit is not, however, in phase with E but lags behind it by an angle $\alpha$ depending upon the self inductance of the circuit and the mutual inductance of the stationary coil. The resistance drop in the potential circuit is $I_p r$ and since this is in phase with $I_p$ and always proportional to it (r being constant), the projection of this resistance drop upon the current $I_s$, or O B, may be considered as representing the reading of the wattmeter. At right angles to this resistance drop is the self-inductive drop of the potential circuit, $I_p x$, and at right angles to the current $I_s$ is the mutual inductive drop $I_s X_m$. The vectorial sum of these three drops is, of course, equal to the impressed voltage E. If the correction factor be defined as the factor by which the wattmeter reading is multiplied to give the true power, then the correction factor in this case is $\frac{O\ A}{O\ B}$, and the problem that presents itself is to find this ratio in terms of the constants of the instrument and load. This correction factor assumes that the instrument has been calibrated on D. C., for in this case $I_p\ r = E$.

Referring to Fig. 20

$$\underline{O\ A} = E \cos \theta$$

$$= E \frac{R}{\sqrt{R^2 + X^2}} \qquad (1)$$

$$\frac{O\,C}{\cdot} = \underline{O\,A} - j\,(\underline{A\,E} - \underline{C\,E}) \qquad (2)$$

$$\underline{A\,E} = E \frac{X}{\sqrt{R^2 + X^2}} \qquad (3)$$

$$\underline{C\,E} = I_s\, X_m$$

$$= \frac{E}{\sqrt{R^2 + X^2}}\, X_m \qquad (4)$$

Substituting (1), (3) and (4) in (2)

$$\frac{O\,C}{\cdot} = \frac{E}{\sqrt{R^2 + X^2}} \left[R - j\,(X - X_m)\right] \qquad (5)$$

Referring again to Fig. 20.

$$I_P^{\,2}\, r^2 + I_P^{\,2}\, x^2 = \overline{O\,C}^{\,2}$$

or

$$I_P = \frac{\underline{O\,C}}{\sqrt{r^2 + x^2}} \qquad (6)$$

From equation (5)

$$\underline{O\,C} = \frac{E}{\sqrt{R^2 + X^2}} \sqrt{R^2 + (X - X_m)^2} \qquad (7)$$

Substituting (7) in (6) and multiplying thru by r

$$I_P\, r = \frac{E\, r \sqrt{R^2 + (X - X_m)^2}}{\sqrt{(r^2 + x^2)(R^2 + X^2)}} \qquad (8)$$

$$\underline{O\,B} = I_P\, r \cos \mathcal{E}$$

$$= I_P\, r \cos (\beta - \Delta) \qquad (9)$$

From equation (5)

$$\beta = \arc \tan \frac{X - X_m}{R} \qquad (10)$$

From figure

$$\Delta = \arc \tan \frac{x}{r} \qquad (11)$$

Substituting (8), (10) and (11) in (9)

$$\underline{O\,B} = \frac{E\, r \sqrt{R^2 + (X - X_m)^2}}{\sqrt{(r^2 + x^2)(R^2 + X^2)}} \cos \left(\arc \tan \frac{X - X_m}{R} - \arc \tan \frac{x}{r}\right) \qquad (12)$$

Whence the correction factor is

$$K = \frac{O\,A}{O\,B} = \frac{R \sqrt{r^2 + x^2}}{r \sqrt{R^2 + (X - X_m)^2}} \sec \left(\arc \tan \frac{X - X_m}{R} - \arc \tan \frac{x}{r}\right) \qquad (13)$$

as obtained by dividing (1) by (12).

By simple trigonometric reduction (13) may be written in the form

$$K = \frac{R\,(r^2 + x^2)}{r\,\left[R\, r + x\,(X - X_m)\right]} \qquad (14)$$

The percent error, referring to Fig. 20, is $\frac{A\,B}{O\,A}$ =

$$\frac{O\,B - O\,A}{O\,A} = \frac{O\,B}{O\,A} - 1.$$

But $\frac{O\,B}{O\,A} = \frac{1}{K}$, and the percent error is $e = \frac{1}{K} - 1$

which by substituting K from equation (14) and reducing becomes

$$e = \frac{r\,x\,(X - X_m) - R\,x^2}{R\,(r^2 + x^2)} \tag{15}$$

An inspection of equations (14) and (15) shows that all the terms contained therein are definite constants of the instrument for a given frequency, except the mutual inductance $X_m$, which depends upon the position of the moving coil, and therefore upon the load. Referring to Fig. 21, let S S be the stationary coils, and M the moving coil, each setting up fluxes as indicated. When the moving coil is in the position A A none of the flux of the stationary coils threads it and consequently in this position $X_m = 0$. When in the position B B the maximum flux from the stationary coils threads it, and so in this position $X_m$ is a maximum. For intermediate positions of the moving coil the variation in this flux threading it is sinusoidal, and there, if $X_M$ denote the maximum value of $X_m$,

$$X_m = X_M \sin \varphi \tag{16}$$

With the coil in the position shown in Fig. 21, the flux of the stationary coil has a component in the same direction as the flux of the moving coil and the sign of $X_m$ is positive; but if it were in the position C C this component thru the moving coil would oppose the flux of the moving coil and the sign of $X_m$ would be negative. If we start from the right angle position of the coils and measure $\varphi$ in the

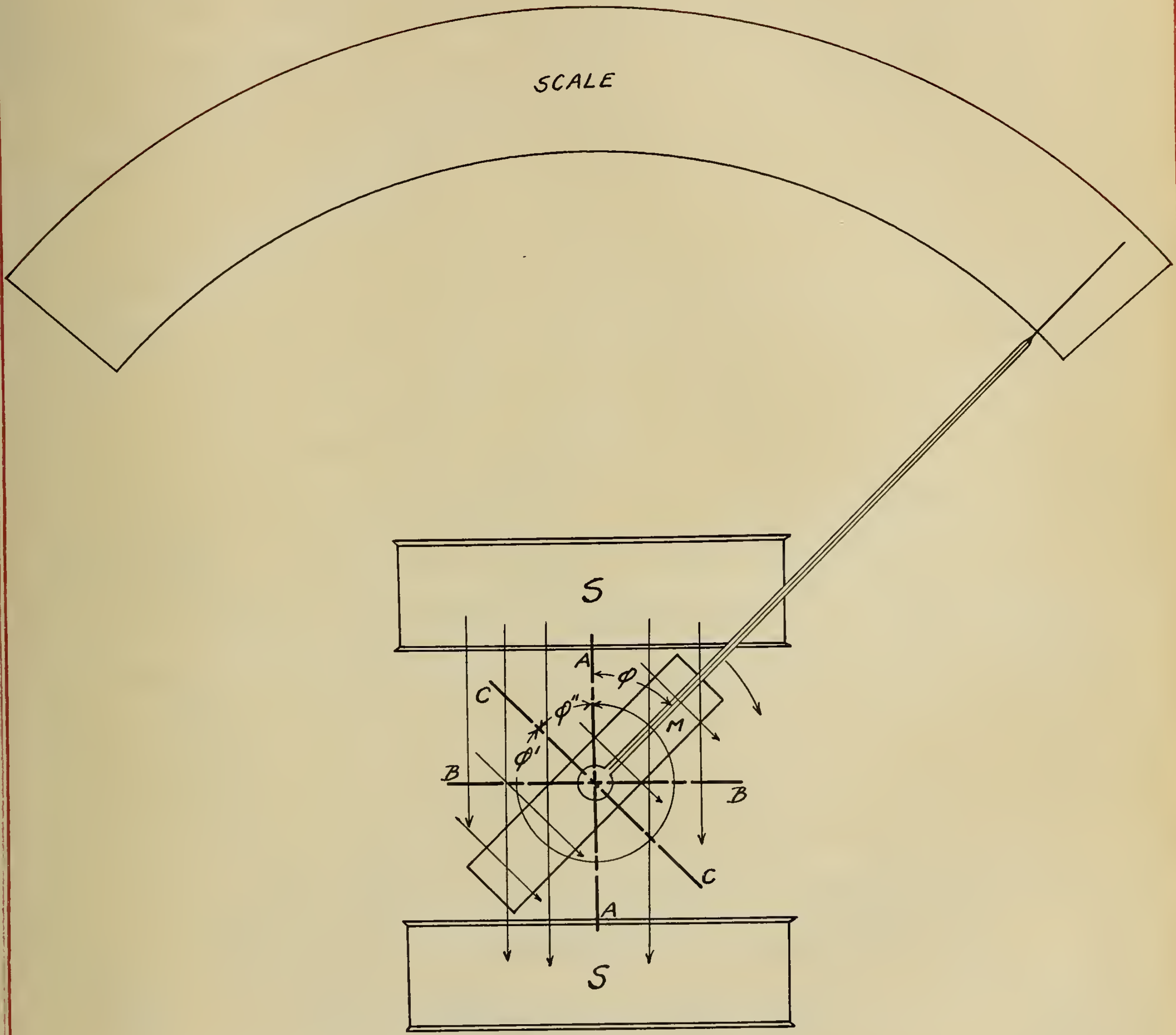



FIG 21.

direction of rotation of the moving coil, it is evident that the change of sign will be taken care of. Thus in the position C C, $\varphi = \varphi' = - \varphi''$. But in a wattmeter the moving coil is not generally visible, and the angle $\varphi$ is not readily measured. It would be convenient, therefore, to have $\varphi$ determined as a function of the wattmeter reading. Such a function will, of course, be different for meters in which the arc thru which the moving coil swings is different, but perhaps, the simplest way to represent it is by plotting it in polar coordinates, where the distance from the pole represents the reading of the meter, and the angle $\varphi$ is read directly. Such a curve may, of course, be plotted from direct physical measurements on any meter, or it may be deduced theoretically. For example, let the coils be arranged as shown in Fig. 22, so that for full scale deflection the coil swings thru 90°, being 45° to the perpendicular at the zero position, and 45° to the perpendicular for full scale reading. Then, taking into account the angular displacement of the moving coil, the magnetic torque is

$$T = \left[K\, I_1\, I_2 \cos\beta\right] \cos\varphi \qquad (17)$$

The resisting torque of the spring is

$$S = k\left(\varphi + \frac{\pi}{4}\right) \qquad (18)$$

When equilibrium is reached, T = S, or

$$\left[K\, I_1\, I_2 \cos\beta\right] \cos\varphi = k\left(\varphi + \frac{\pi}{4}\right) \qquad (19)$$

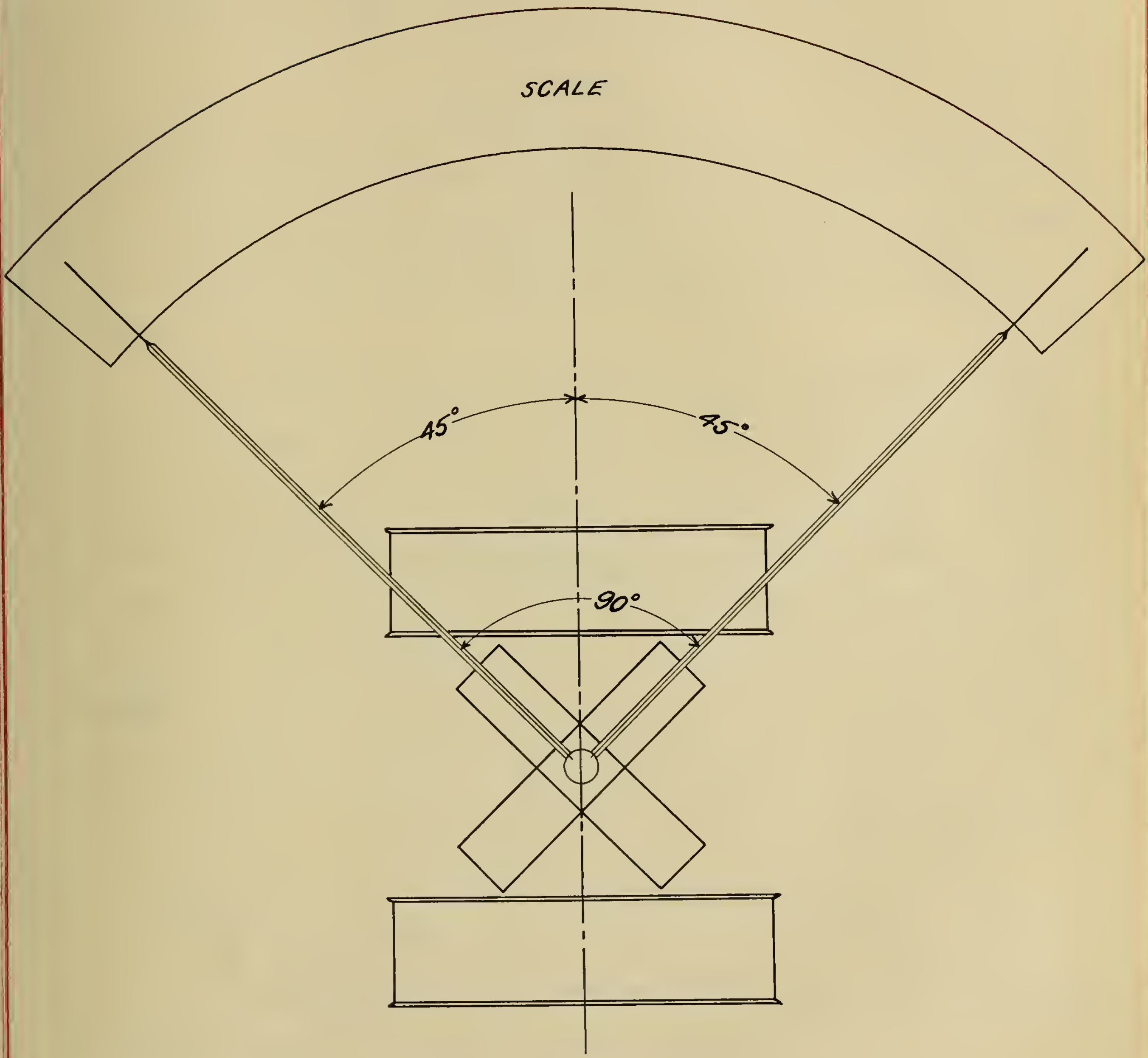



FIG. 22.

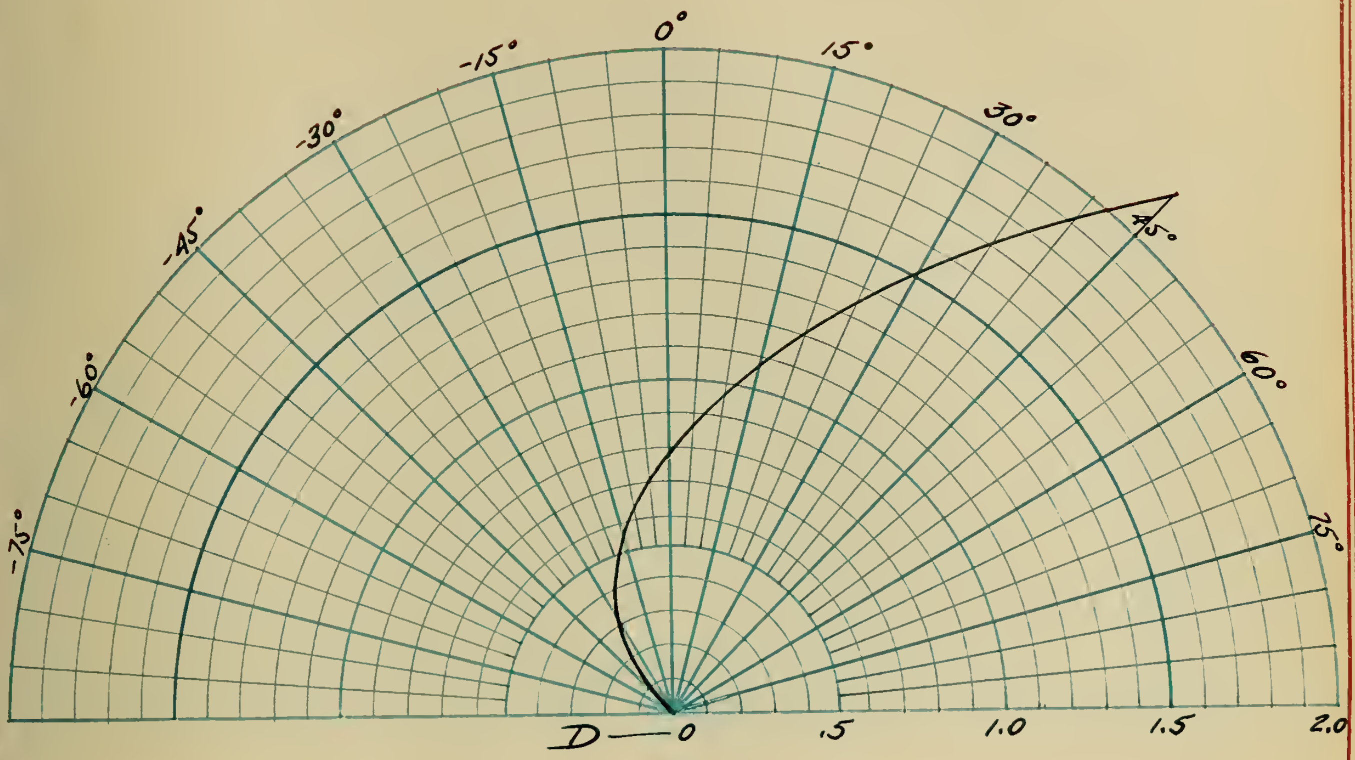
$\left[ K\ I_1\ I_2 \cos \beta \right]$ is the reading of the meter and will be denoted by D. Then (19) may be written

$$D = \frac{k}{\cos \varphi} \left(\varphi + \frac{\pi}{4}\right) \tag{20}$$

which gives the relation between D and $\varphi$ in an instrument in which the coils are arranged as shown in Fig. 22. The graph of equation (20), with k equal to unity is given on Curve Sheet 2.

If the reading of the meter for full-scale deflection is known, the value of k in equation (20) may be solved for by substituting this value for D and $\frac{\pi}{4}$ for $\varphi$. Then from a plotted curve, similar to that on Curve Sheet 2, the value of $\varphi$ for any reading may be obtained, which value may be substituted in equation (16), and $X_m$ solved for if $X_M$ is known. $X_M$ is the mutual inductive reactance when the coils are parallel, and may be determined experimentally or by calculation. A further shortening of the process would be brought about if a curve were plotted between $X_m$ and D, from which the value of $X_m$ could be taken directly. Such a curve, corresponding to the one on Curve Sheet 2 and with $X_M$ taken as unity, is shown on Curve Sheet 3.

The preceding discussion and formulae apply to a non-compensated meter connected as shown in Fig. 18-b as well as to a compensated meter, except that with the connection shown in Fig. 18-b, R and X must include the resistance and reactance of the series coil, and the further correction must be made of subtracting the $I^2 R$ loss in the series coil.




CURVE SHEET 2

SHOWING VARIATION OF

ANGULAR POSITION OF MOVING COIL

WITH

READING OF METER.


CURVE SHEET 3
SHOWING
VARIATION OF
MUTUAL REACTANCE
WITH
READING OF A WATTMETER.

In this case, then, the true load is

$$W = \frac{R'\ (r^2 + x^2)}{r\ \left[R'\ r + x\ (X' - X_m)\right]}\ D - I^2\ R_s \qquad (21)$$

where $R' = (R + R_s)$, $X' = (X + X_s)$, and I is the load current.

In order that an idea of the magnitude of these errors and corrections under various conditions may be obtained it will be well to consider the following examples. Let the wattmeters have the arrangement of stationary and moving coils as given in Fig. 22, and assume the constants of the instrument to be as given.

EXAMPLE I.

Compensated Meter.

Capacity -- 0-75 watts

Potential - 0-75 volts

Maximum current -- 4 amp.

$R_s$ = .03 ohms

$L_s$ = .0003 henries

$X_s$ = .1131 ohms at 60 cycles

$L_M$ = .0002 henries

$X_M$ = .0753 ohms at 60 cycles

r = 1500 ohms

l = .0007 henries

x = .264 ohms at 60 cycles

It will be remembered that the error is greatest at low power-factor. Let the meter be considered as operating

at 60 cycles, on a load of .1 power-factor, and study the variation of the percent error with the load at various voltages, say, 75, 40, and 10 volts.

From equation (15)

$$e = \frac{396\ (X - X_m) - .07\ R}{2,250,000.07\ R}$$

$$= \frac{396\ \dfrac{X - X_m)}{R} - .07}{2,250,000} \qquad (22)$$

At .1 power-factor $\frac{X}{R}$ = 9.95, whence X may be obtained for various values of R, as may also the load.

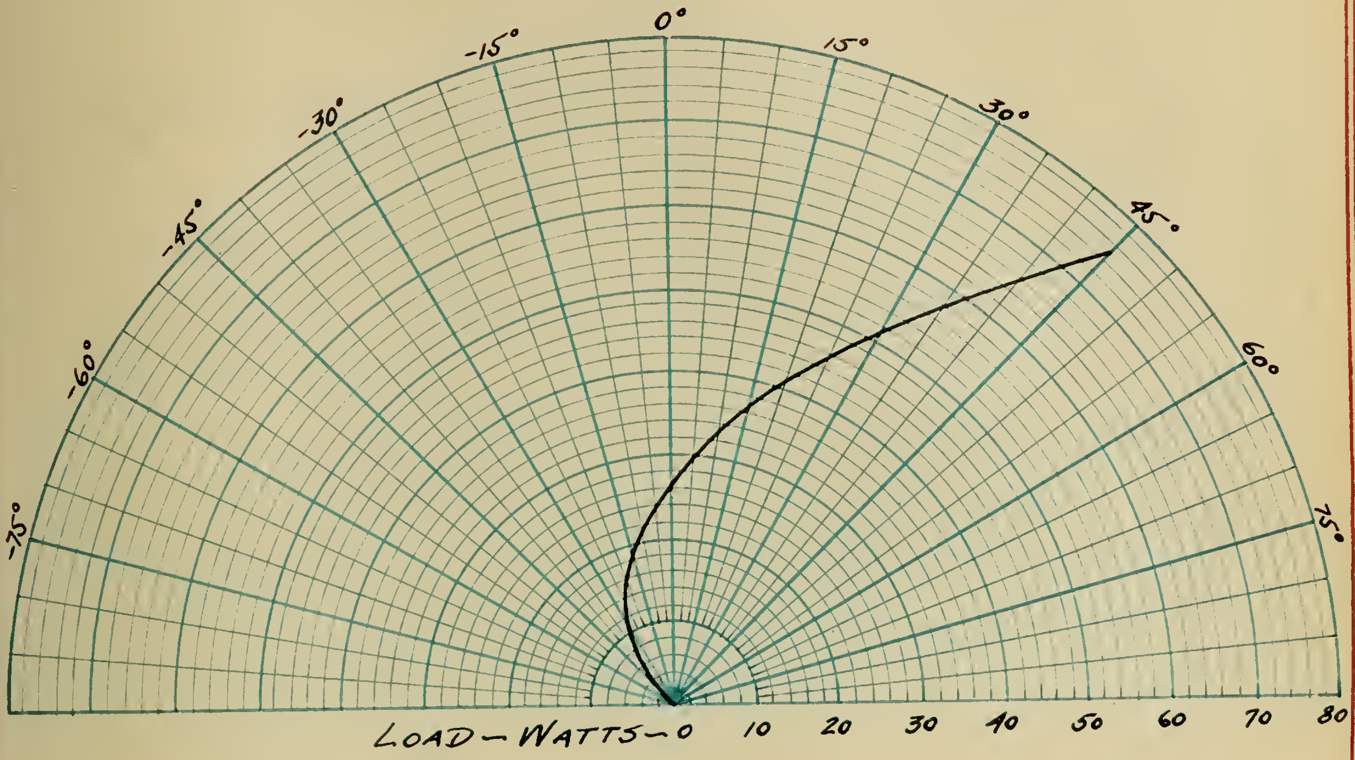
To determine $X_m$, let D = 75 in equation (20), and $\varphi = \frac{\pi}{4}$. Then, using degrees rather than radians as being more convenient and equally well suited in this case for the designation of angles,

$$75 = \frac{k}{\cos 45°}\ (45 + 45) = \frac{90\ k}{.707}$$

or $$k = \frac{.707 \times 75}{90} = .589$$

and $$D = \frac{.589}{\cos \varphi'}\ (\varphi' + 45)$$

where $\varphi'$ is expressed in degrees. On Curve Sheet 4 is given the graph of this expression, from which $\varphi$ may be obtained for any reading of the wattmeter, and since the discrepency between the reading and the true load is for this purpose comparatively negligible, we may say that $\varphi$ for any load is




CURVE SHEET 4
SHOWING VARIATION OF
ANGULAR POSITION OF MOVING COIL
WITH
LOAD.

obtained from this curve. Then from the expression

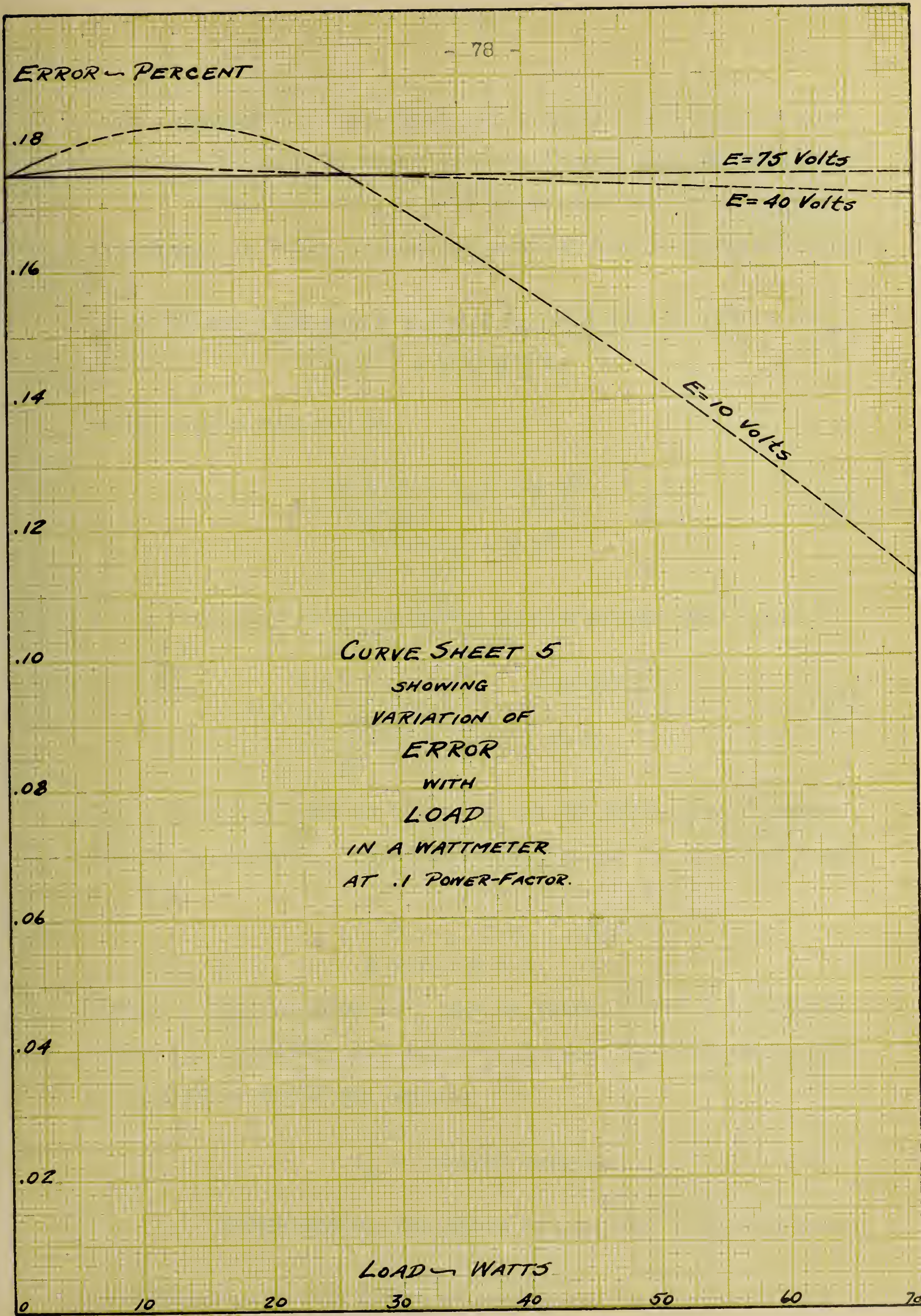
$$X_m = .0753 \sin \varphi$$

$X_m$ for any load is obtained.

Substituting the values of R, X and $X_m$ thus obtained in equation (22) the percent error is obtained for the load corresponding to these values of X, R, and impressed voltage.

On Curve Sheet 5 are drawn three curves between load and percent error, one at 75 volts, one at 40 volts, and one at 10 volts. The full line part of the curve represents the part over which the meter may operate, but on the dotted line part the current thru the stationary coil exceeds the allowable maximum of 4 amperes, and so is not a safe region over which to operate the meter. At 75 volts the percent error is practically constant at .175%, X being so great as compared with $X_m$ as to make the effect of the latter practically neglige. At 40 volts the percent error is slightly increased within the operative range, and decreased beyond the point where $X_m$ changes sign. At 10 volts this increase and decrease in the percent error are still more marked. In all cases, however, the percent error in the operative range lies within the narrow limits of .175% and .180%.

It is sometimes desired to measure power at a frequency higher than commercial frequencies, and it is interesting to know what degree of accuracy may be expected


ERROR — PERCENT
.18
.16
.14
.12
.10
.08
.06
.04
.02
E=75 Volts
E=40 Volts
E=10 Volts
CURVE SHEET 5
SHOWING
VARIATION OF
ERROR
WITH
LOAD
IN A WATTMETER
AT .1 POWER-FACTOR.
LOAD — WATTS
0
10
20
30
40
50
60
70

from a commercial instrument under these conditions. The variation of percent error with frequency will be considered.

Equation (15) may be rewritten, so as to express inductance in terms of henries and frequency instead of ohms, thus

$$e = \frac{r\ (2\ \pi\ f\ l)(2\ \pi\ f\ L - 2\ \pi\ f\ L_m) - R\ (2\ \pi\ f\ l)^2}{R\ (r^2 + \overline{2\ \pi\ f\ l}^{\,2})} \qquad (23)$$

which may be reduced to

$$e = \frac{r\ l\ (L - L_m) - R\ l^2}{\dfrac{R\ r^2}{(2\ \pi\ f)^2} + R\ l^2} \qquad (24)$$

For the particular meter under consideration (24) becomes

$$e = \frac{1.05\ (L - L_m) - .00000049\ R}{\dfrac{57{,}100\ R}{f^2} + .000000\ 49\ R} \qquad (25)$$

Let a constant load of 50 watts, at 50 volts and .8 power-factor be assumed. This determines R and X; R = 3.2 and X = 2.4.

$$L = \frac{X}{2\ \pi\ f} = \frac{2.4}{2\ \pi\ f} = \frac{.382}{f}$$

whence L for any frequency may be found. $L_m$ has a constant value for 50 watts, and may be obtained by dividing $X_m$ at 60 cycles by 2 π x 60, whence

$$L_m = .000097 \text{ henries}$$

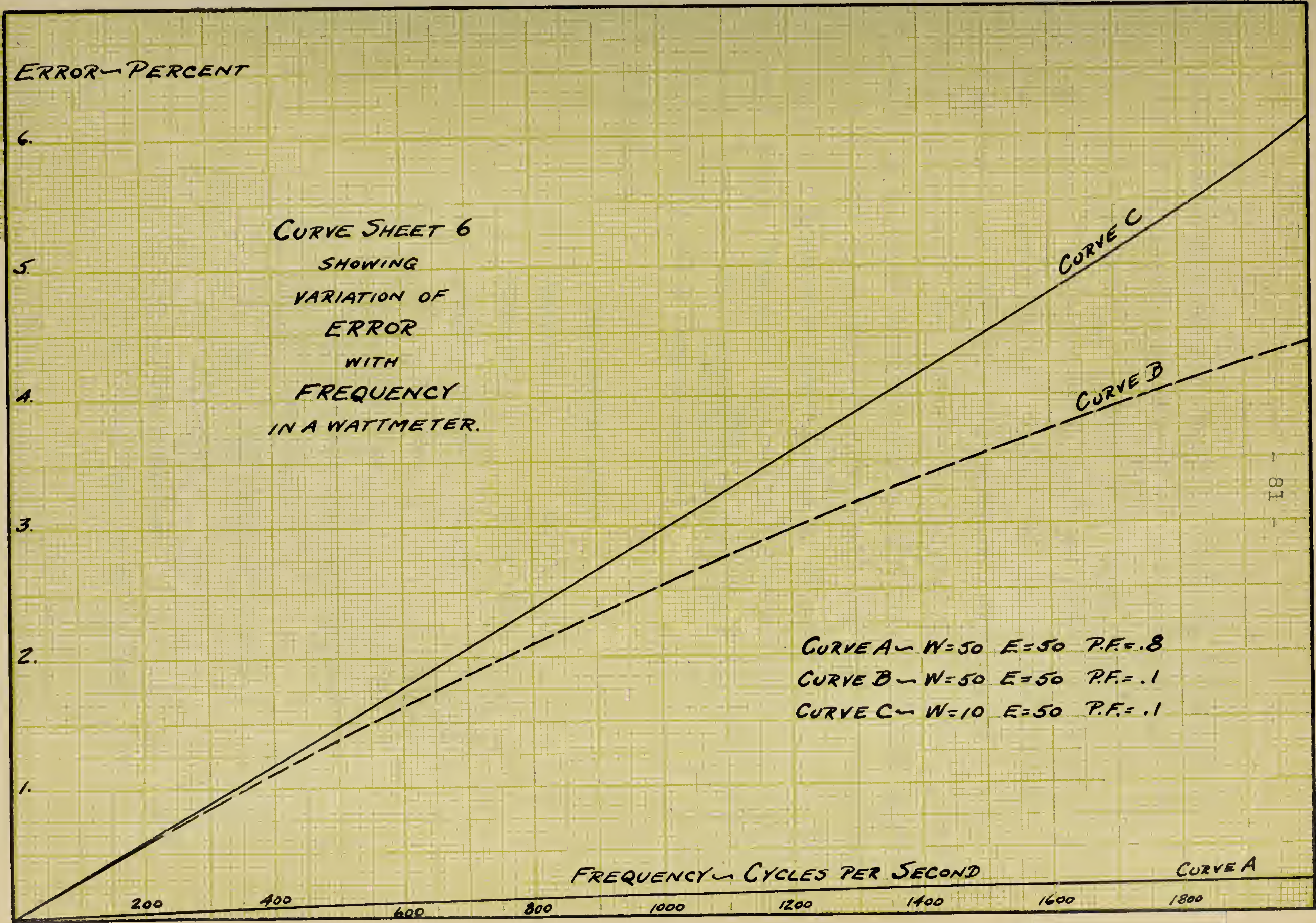
Substituting the above values for L, $L_m$ and R in

(25), the percent error becomes

$$e = \frac{1.05\left(\frac{.382}{f} - .000097\right) - .00000157}{\frac{182,900}{f^2} + .00000157}$$

which gives the variation of the percent error with the frequency. Curve A, on Curve Sheet 6 represents this variation.

Similarly, a curve may be plotted showing the variation of percent error with the frequency at the same load and voltage but at .1 power-factor. The dotted line curve B on Curve Sheet 6 represents this variation. At .1 power-factor 50 watts at 50 volts is beyond the range of the current coil, and hence Curve B is drawn dotted. Curve C on the same sheet shows the variation of percent error with frequency at 10 watts, 50 volts, and .1 power-factor. B and C curve in opposite directions due to the difference in the sign of $L_m$ at 50 watts and 10 watts. Even at 2,000 cycles it is seen that the error at .8 power-factor is less than .25%. For a power-factor of .1, however, the error is in the neighborhood of 5% at 2000 cycles, showing a very great increase in error at low power-factors, as was to be expected from previous considerations. The variation of percent error with power-factor will be taken up next. A load of 50 watts at 50 volts and a frequency of 1000 cycles is assumed. The power-factor is varied from .1 lagging current, thru unity to .1 leading current. At 50 watts and


ERROR ~ PERCENT
6.
5.
4.
3.
2.
1.
CURVE SHEET 6
SHOWING
VARIATION OF
ERROR
WITH
FREQUENCY
IN A WATTMETER.
CURVE C
CURVE B
CURVE A
CURVE A ~ W=50 E=50 P.F.=.8
CURVE B ~ W=50 E=50 P.F.=.1
CURVE C ~ W=10 E=50 P.F.=.1
FREQUENCY ~ CYCLES PER SECOND
200
400
600
800
1000
1200
1400
1600
1800

1000 cycles there is a definite value of $X_m$ which may be found as previously indicated. For each power-factor there is a definite ratio $\frac{X}{R}$, X being considered negative for leading current; and at 50 watts there is a definite value of R for a given power-factor. Hence, for a given power-factor, X may be found, and substituting these values together with the constants of the instrument in equation (15), the percent error at the given power factor is obtained. On Curve Sheet 7 is a curve which was thus calculated, showing the variation of percent error with power-factor at 50 watts, 50 volts, and 1000 cycles. The error is seen to increase rapidly at low power-factors, the increase being more rapid for a leading current than for a lagging current.

EXAMPLE II.

Compensated Meter

| | |
|---|---|
| Capacity | 0-7.5 watts |
| Potential | 0-15 volts |
| Maximum current | 2 amp. |

$R_s$ = .12 ohms

$L_s$ = .0012 henries

$X_s$ = .4525 ohms at 60 cycles

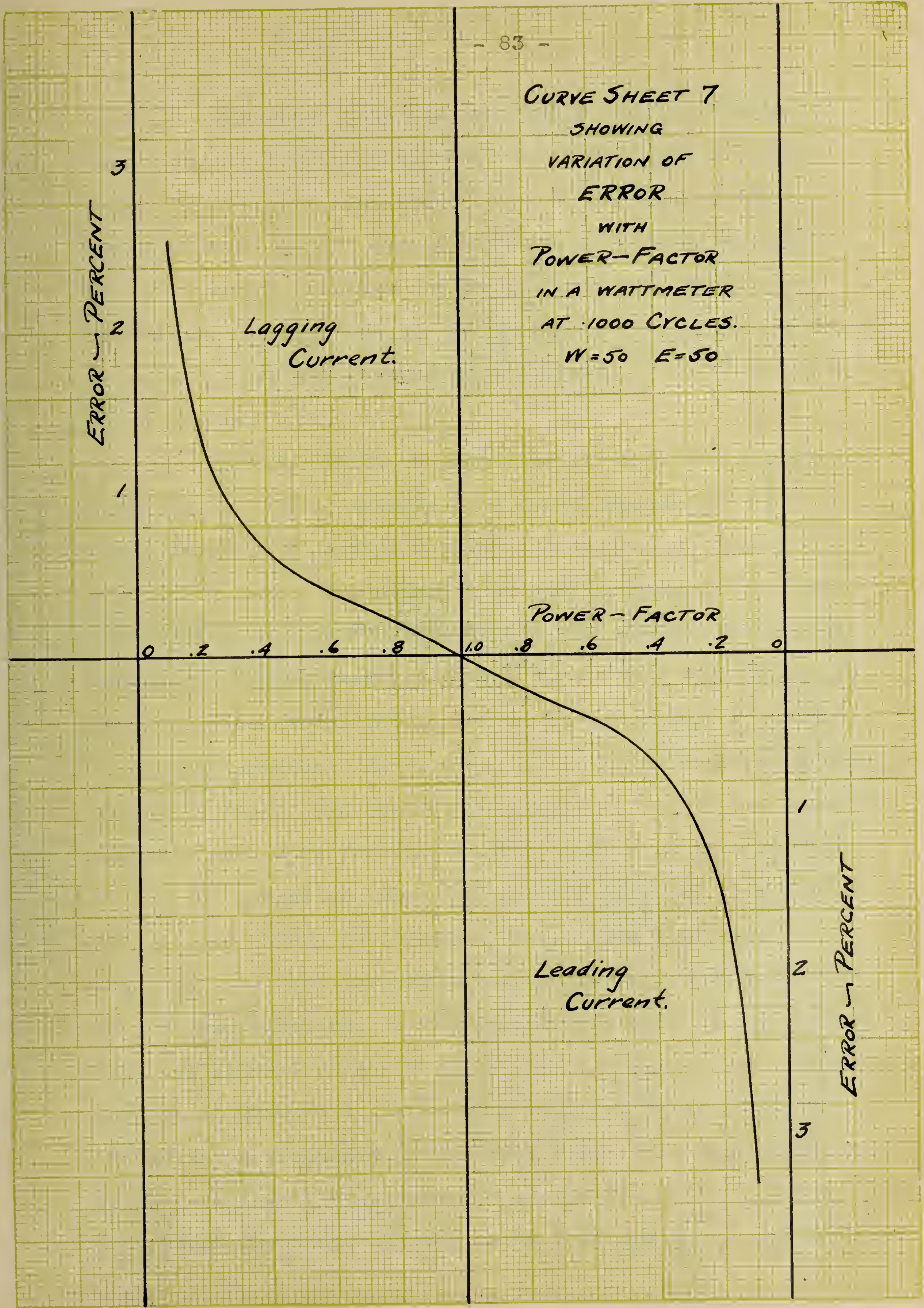
$L_M$ = .0006 henries

$X_M$ = .2259 ohms at 60 cycles

r = 300 ohms

l = .0007 henries

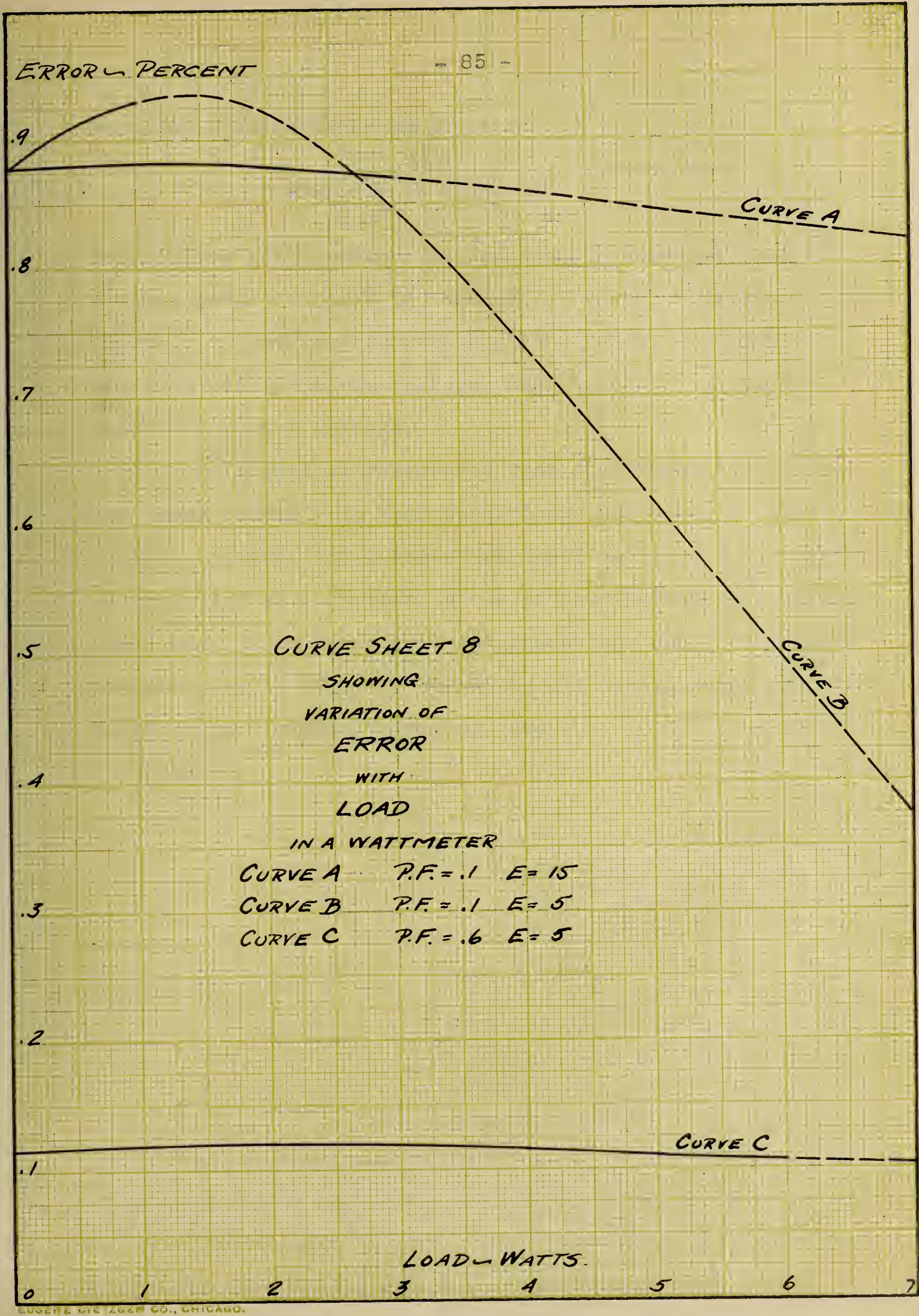
x = .264 ohms at 60 cycles



Curve Sheet 7
showing
variation of
error
with
power-factor
in a wattmeter
at 1000 cycles.
W=50 E=50
Lagging
Current.
Leading
Current.
Power-Factor
0 .2 .4 .6 .8 1.0 .8 .6 .4 .2 0
Error - Percent
1
2
3
Error - Percent
1
2
3

The above meter is similar to the one considered under Example 1, with the exception that it has but one-tenth the capacity, and has a lower voltage range and a lower current range. Curves similar to those given under Example 1 might be given for this meter, but it will be sufficient for purposes of comparison to plot only one set of curves, viz., the variation of percent error with loads at 60 cycles.

Assume, first, a power-factor of .1 and a potential of 15 volts. As in Example 1, a curve represented by A on Curve Sheet 8 is plotted showing the variation of percent error with load. A similar curve for .1 power-factor, but for 5 volts instead of 15 volts, is represented by B on the same Curve Sheet. Curve C on the same sheet represents the same variation for 5 volts, but at .6 power-factor instead of .1 power-factor. As before, the full line represents the operative range, and the dotted line that part where excessive current flows thru the series coil. Again it is seen that within the operative range the percent error at .1 power-factor lies within the comparatively close limits of .87% and .93%, and that the percent error is very much increased at low power-factors. A comparison with Curve Sheet 5 shows that the percent error is about five times as great in this case as in Example 1, from which it might be expected that in similar instruments the percent error will vary about inversely as the voltage rating. An inspection of equation (15) bears this out. If the terms $R\,x^2$ and $X_m$ be neglected


ERROR ~ PERCENT
.9
.8
.7
.6
.5
.4
.3
.2
.1
CURVE A
CURVE B
CURVE C
CURVE SHEET 8
SHOWING
VARIATION OF
ERROR
WITH
LOAD
IN A WATTMETER
CURVE A P.F. = .1 E = 15
CURVE B P.F. = .1 E = 5
CURVE C P.F. = .6 E = 5
LOAD ~ WATTS
0
1
2
3
4
5
6
7

as being comparatively small, then

$$e = \frac{r \times X}{R\, r^2} = x \cdot \frac{X}{R} \cdot \frac{1}{r}$$

For similar instruments x is practically constant, and for a given power-factor $\frac{X}{R}$ is constant. Whence $e$ varies directly as $\frac{1}{r}$. But the resistance of the potential circuit, r, varies directly as the voltage, and therefore, $e$ varies inversely as the voltage.

It will be remembered that the errors considered are those peculiar to load conditions, and apply to compensated meters. In a non-compensated meter, the additional error due to $I^2R$ loss in one of the instrument windings would have to be taken into consideration, and in all there are the sources of error common to all instruments, as temperature, friction parallax, etc.

# CHAPTER IV

## INSTRUMENT TRANSFORMERS

On high potential alternating current systems it is often desirable to obtain energy for the operation of metering and regulating apparatus by the use of potential transformers to step down the voltage, and current transformers to provide a current circuit not metallically connected to the high potential system. These instrument transformers contain iron as an essential element, and are therefore, somewhat evasive to strict theoretical treatment under conditions where the flux in the iron core varies. In the case of potential transformers this variation is negligible when the transformer is used on practically constant voltage, but in the case of current transformers where the flux varies with the main load current and the transformer is called upon to operate over a considerable current range, the saturation curve of the iron must be taken into account and the treatment becomes somewhat involved. A very brief consideration of the current transformer will be given here, and for a more complete discussion reference is made to Bulletin No. 60 , of the Engineering Experiment Station of the University of Illinois entitled "The Characteristics and Limitations of the Series Transformer."

A vector diagram of the operation of the current transformer is given in Fig. 23, where the secondary current $I_2$ is taken as reference vector. The induced secondary e.m.f.

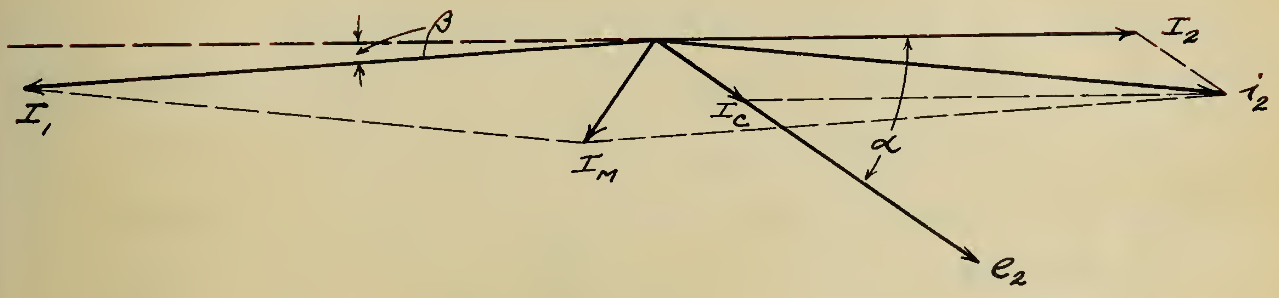



FIG. 23.

$e_2$, leads $I_2$ by an angle $\alpha$ depending upon the total resistance and reactance (including leakage) of the secondary circuit. Core loss may be represented by a component of current in phase with the induced secondary voltage, as $I_c$. The vectorial sum of $I_2$ and $I_c$ gives the virtual secondary current $i_2$. The magnetizing current $I_M$ leads $e_2$ by 90°, and is the sum of the virtual secondary current $i_2$ and primary current $I_1$.

The core loss varies as a power of the flux density somewhere between 1.6 and 2.0, and the flux density varies directly as the induced e.m.f. $e_2$. Hence, the core loss varies as a power of the induced e.m.f. somewhere between 1.6 and 2.0. It is assumed to vary as the square of the induced e.m.f. whence the core loss may be expressed as

$$W_c = \frac{e_2^{\,2}}{R_c} = \left(\frac{e_2}{R_c}\right) e_2 \qquad (1)$$

$R_c$ is the effective core loss resistance, and $\left(\frac{e_2}{R_c}\right)$ is the core loss current in phase with $e_2$, or

$$\dot{I}_c = \frac{\dot{e}_2}{R_c} \qquad (2)$$

But

$$\dot{e}_2 = I_2 \,(R - j\,X) \qquad (3)$$

where R and X are the total secondary resistance and reactance respectively. Substituting (3) in (2),

$$\dot{I}_c = \frac{I_2}{R_c}\,(R - j\,X) \qquad (4)$$

The flux in the core is

$$\dot{\varphi}_M = \frac{.4\,\pi}{\rho}\; N_2\, \dot{I}_M \qquad (5)$$

where $\rho$ is the reluctance of the core, and $N_2$ is the number of secondary turns:

Also,

$$\dot{\varphi}_M = \frac{\dot{e}_2 \times 10^8}{2\,\pi\, f\, N_2}\,(-\, j) \qquad (6)$$

Substituting (3) in (6)

$$\dot{\varphi}_M = -\,\frac{I_2 \times 10^8}{2\,\pi\, f\, N_2}\,(X + j\, R) \qquad (7)$$

Equating (5) and (7)

$$\dot{I}_M = -\,\frac{I_2}{(2\,\pi\, f)\,\frac{.4\,\pi}{\rho}\, N_2^{\,2} \times 10^{-8}}\,(X + j\, R) \qquad (8)$$

But

$$(2\,\pi\, f)\,\frac{.4\,\pi}{\rho}\, N_2^{\,2} \times 10^{-8} = X_2 \qquad (9)$$

where $X_2$ is the reactance of the secondary due to the flux in the iron core alone.

Substituting (9) in (8)

$$\dot{I}_M = -\,\frac{I_2}{X_2}\,(X + j\, R) \qquad (10)$$

$$N_1\, \dot{I}_1 + N_2\, \dot{i}_2 = N_2\, \dot{I}_M \qquad (11)$$

$$\dot{i}_2 = \dot{I}_2 + \dot{I}_c \qquad (12)$$

Substituting (4) in (12)

$$i_2 = \frac{I_2}{R_c} \left[ (R_c + R) - j\,X \right] \tag{13}$$

Substituting (10) and (13) in (11)

$$N_1\, I_1 + \frac{N_2\, I_2}{R_c} \left[ (R_c + R) - j\,X \right] = - \frac{N_2\, I_2}{X_2} (X + j\,R) \tag{14}$$

which reduces to

$$I_1 = - \frac{N_2}{N_1} \left[ \left(1 + \frac{R}{R_c} + \frac{X}{X_2}\right) + j \left(\frac{R}{X_2} - \frac{X}{R_c}\right) \right] I_2 \tag{15}$$

Using the following abbreviations

$$A = \left(1 + \frac{R}{R_c} + \frac{X}{X_2}\right) \tag{16}$$

$$B = \left(\frac{R}{X_2} - \frac{X}{R_c}\right) \tag{17}$$

the current ratio obtained from (15) is

$$\frac{I_2}{I_1} = - \frac{N_1}{N_2} \frac{1}{\sqrt{A^2 + B^2}} \tag{18}$$

and the phase angle is

$$\beta = \text{arc tan } \frac{B}{A} \tag{19}$$

If tan $\beta$ is positive, or since A is always positive, if B is positive, then $\beta$ lies in the third quadrant and the secondary current leads the 180° position with reference to the primary current. For this condition (See (17)), $\frac{R}{X_2}$ must be greater than $\frac{X}{R_c}$, and since this is generally the case, the secondary current generally leads the 180° position. But with sufficiently high core loss, high secondary reactance, low secondary resistance and a large number of secondary turns, thus making $X_2$ large;

$\frac{R}{X_2}$ may be less than $\frac{X}{R_c}$ and the phase angle go from leading to lagging.

For a given secondary load R and X remain constant at a given frequency, $R_c$ was assumed constant, and $X_2$ varies very greatly with load conditions depending upon the saturation curve of the iron. It will suffice here to assume a transformer and secondary load with given constants at normal full load, and calculate the current ratio, and phase angle.

Let the constants be as follows:

$$R = 1 \text{ ohm}$$

$$X = 5 \text{ ohms at 60 cycles}$$

$$R_c = 1000 \text{ ohms}$$

$$X_2 = 95 \text{ ohms at 60 cycles}$$

$$N_1 = 14$$

$$N_2 = 133$$

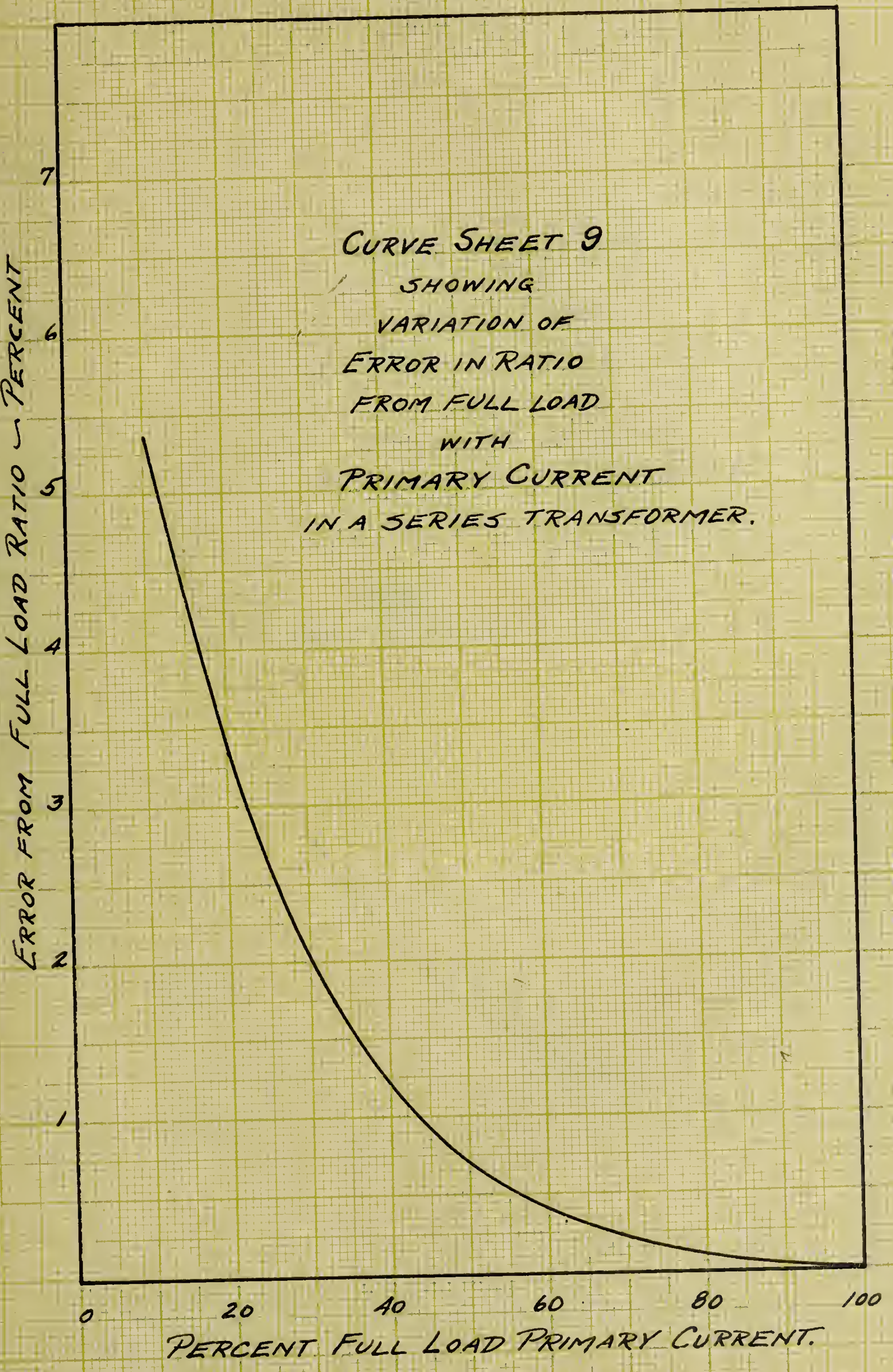
$$A = 1 + \frac{1}{1000} + \frac{5}{95} = 1.0536$$

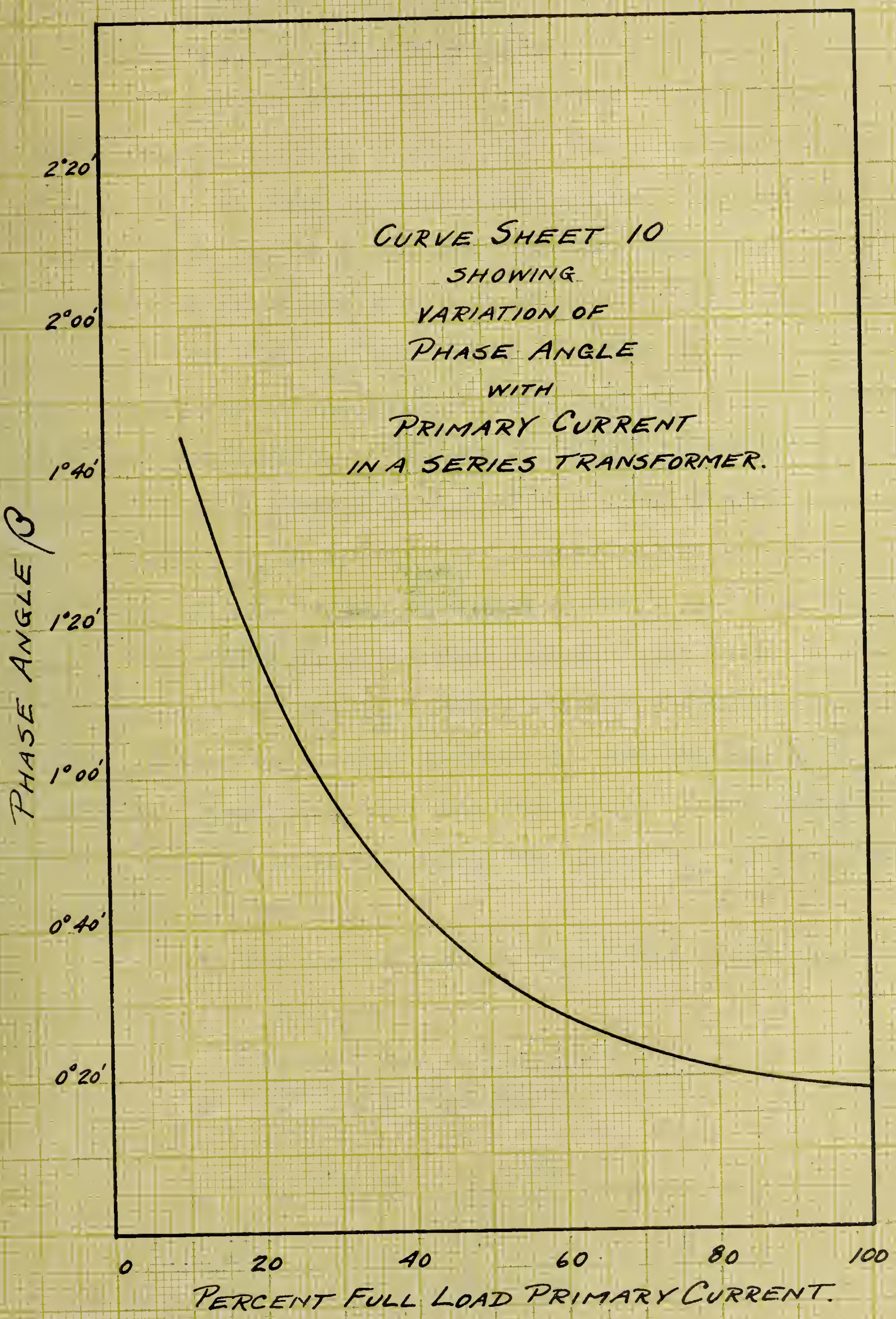
$$B = \frac{1}{95} - \frac{5}{1000} = .01053 - .005 = .00553$$

$$\frac{I_2}{I_1} = - \frac{14}{133\sqrt{\overline{1.0536}^2 + \overline{.00553}^2}} = .1$$

$$\beta = \text{arc tan } \frac{.00553}{1.0536} = \text{arc tan } .00525 = 18' \text{ (leading)}.$$

On Curve Sheet 9 is given a typical curve showing the variation of percent error from full load current-ratio with primary current. Such a curve, of course, depends upon the saturation curve of the iron, and must be determined


CURVE SHEET 9
SHOWING
VARIATION OF
ERROR IN RATIO
FROM FULL LOAD
WITH
PRIMARY CURRENT
IN A SERIES TRANSFORMER.
ERROR FROM FULL LOAD RATIO ~ PERCENT
1
2
3
4
5
6
7
0
20
40
60
80
100
PERCENT FULL LOAD PRIMARY CURRENT.


CURVE SHEET 10
SHOWING
VARIATION OF
PHASE ANGLE
WITH
PRIMARY CURRENT
IN A SERIES TRANSFORMER.
PHASE ANGLE β
2°20'
2°00'
1°40'
1°20'
1°00'
0°40'
0°20'
0
20
40
60
80
100
PERCENT FULL LOAD PRIMARY CURRENT.

experimentally. On Curve Sheet 10 is a similar curve showing a typical variation of phase angle with primary current.

The operation of the potential transformer is shown by the vector diagram in Fig. 24. The secondary terminal e.m.f., $E_2$, is taken as reference vector. The secondary current $I_2$ lags behind $E_2$ an angle $\alpha$ depending upon the resistance R and reactance X of the secondary load. The induced secondary voltage $e_2$ leads $E_2$ by an angle $\beta$ depending upon the resistance of the secondary winding $r_2$, and the secondary leakage reactance $x_2$. Leading $e_2$ by 90° is the flux in the core $\varphi_M$ and in phase with this flux is the magnetizing current $I_M$. Leading $I_M$ by 90° is the primary inducing voltage $e_1$ and in phase with $e_1$ is the core loss current $I_c$. The vectorial sum of the magnetizing current and the core loss current is the exciting current $I_E$. The vectorial difference between $I_2$ and $I_E$ gives the primary current $I_1$, and leading $I_1$ by an angle $\gamma$, depending upon the resistance of the primary winding $r_1$ and the primary leakage reactance $x_1$, is the primary impressed voltage $E_1$.

By the use of complex quantities, the diagram may be analyzed as follows:

$$\underset{\cdot}{I}_2 = \frac{E_2}{Z} = \frac{E_2}{R - j\,X} = \frac{E_2}{Z^2}\,(R + j\,X) \qquad (20)$$

$$\underset{\cdot}{e}_2 = E_2 + \underset{\cdot}{I}_2\,Z_2 = E_2 + \frac{E_2}{Z^2}\,(R + j\,X)(r_2 - j\,x_2)$$

$$= \frac{E_2}{Z^2}\,(A + j\,B) \qquad (21)$$

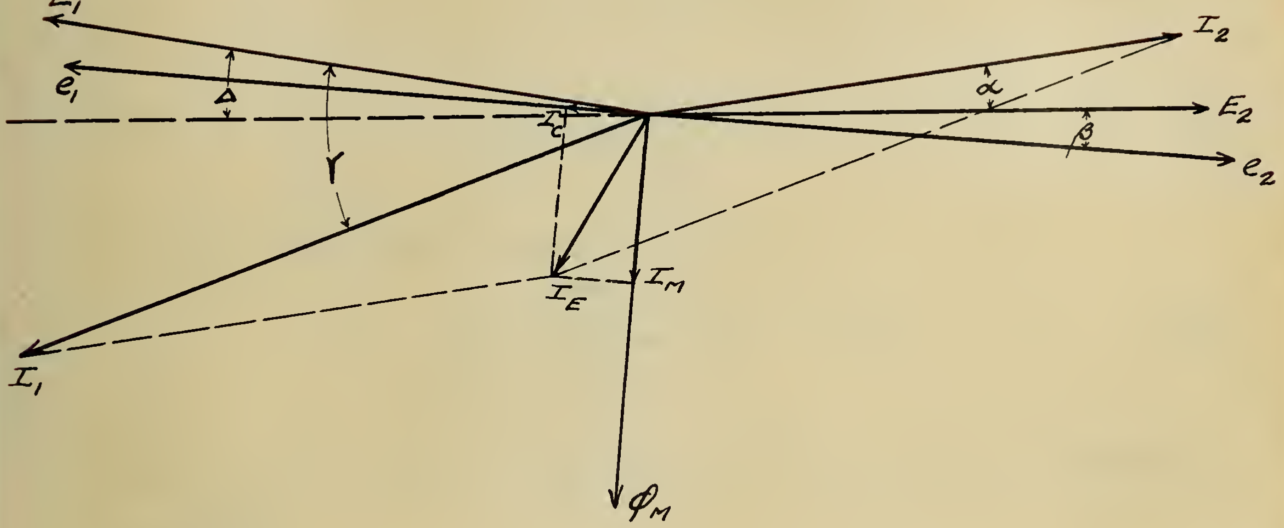



FIG. 24.

where $$A = Z^2 + R\,r_2 + X\,x_2$$

and $$B = X\,r_2 - R\,x_2$$

$$\dot{e}_1 = -\frac{N_1}{N_2}\,\dot{e}_2 = -\frac{N_1}{N_2}\,\frac{E_2}{Z^2}\,(A + j\,B) \tag{22}$$

where $N_1$ and $N_2$ are the primary and secondary turns respectively.

$$\dot{\Phi}_M = \frac{\dot{e}_2 \times 10^8}{2\,\pi\,f\,N_2}\,(-\,j) = \frac{E_2 \times 10^8}{Z^2 \times 2\,\pi\,f\,N_2}\,\left[B - j\,A\right] \tag{23}$$

Also $$\dot{\Phi}_M = \frac{.4\,\pi\,N_1\,\dot{I}_M}{\rho} \tag{24}$$

Equating (23) and (24) and reducing,

$$\dot{I}_M = \frac{E_2}{Z^2\,X_M}\,\left[B - j\,A\right] \tag{25}$$

where $X_M = 2\,\pi\,f \times \frac{.4\,\pi\,N_1\,N_2}{\rho} \times 10^{-8}$, the mutual inductive reactance.

As in the case of the current transformer, the core loss is assumed to vary as the square of the flux density, and

$$\dot{I}_c = \frac{\dot{e}_1}{R_c} = -\frac{N_1}{N_2}\,\frac{E_2}{Z^2\,R_c}\,\left[A + j\,B\right] \tag{26}$$

$$\dot{I}_E = \dot{I}_M + \dot{I}_c$$

$$= \frac{E_2}{Z^2}\left[\left(\frac{B}{X_M} - \frac{N_1}{N_2}\,\frac{A}{R_c}\right) - j\left(\frac{A}{X_M} + \frac{N_1}{N_2}\,\frac{B}{R_c}\right)\right] \tag{27}$$

$$N_1\,\dot{I}_1 + N_2\,\dot{I}_2 = N_1\,\dot{I}_E$$

Substituting (20) and (27) in (28), and reducing

$$\dot{I}_1 = \frac{E_2}{Z^2} \left[ C - j\, D \right] \tag{28}$$

where

$$C = \left(\frac{B}{X_M} - \frac{N_1\, A}{N_2\, R_c} - \frac{N_2\, R}{N_1}\right)$$

and

$$D = \left(\frac{A}{X_M} + \frac{N_1}{N_2}\frac{B}{R_c} + \frac{N_2\, X}{N_1}\right)$$

$$\dot{E}_1 = \dot{e}_1 + \dot{I}_1\, \dot{Z}_1$$

$$= - \frac{N_1\, E_2}{N_2\, Z^2} (A + j\, B) + \frac{E_2}{Z^2} (C - j\, D)(r_1 - j\, x_1)$$

$$= - \frac{N_1}{N_2} \frac{E_2}{Z^2} \left[ G + j\, H \right] \tag{29}$$

where

$$G = (A - \frac{N_2}{N_1} C\, r_1 + \frac{N_2}{N_1} D\, x_1)$$

and

$$H = (B + \frac{N_2}{N_1} C\, x_1 + \frac{N_2}{N_1} D\, r_1)$$

Whence the voltage ratio is

$$\frac{E_2}{E_1} = - \frac{N_2}{N_1} \frac{Z^2}{\sqrt{G^2 + H^2}} \tag{30}$$

and the phase angle is

$$\Delta = \text{arc tan} \frac{H}{G} \tag{31}$$

The factor G is always positive. If the factor H

is negative, as is generally the case, $\Delta$ lies in the second quadrant and the secondary voltage lags behind the 180° position with reference to the primary voltage. But if H is positive as it may be with large secondary load reactance, low leakage, high core reluctance, and low core loss, $\Delta$ will be in the third quadrant, and the secondary voltage will lead the 180° position.

A variation in voltage or frequency, unless both increase or decrease in the same proportion, means a variation in flux density, and a consequent variation in $R_c$ and $X_M$. As before, $R_c$ may be assumed constant, but the variation in $X_M$ is too great to warrant such an assumption, and therefore use must be made of the saturation curve of the iron. Knowing that $X_M$ varies directly as the permeability of the iron, and that for a given voltage and frequency the flux and consequently the point of operation on the saturation curve may be found, $X_M$ for any voltage and frequency may be determined. But potential transformers are not generally called upon to operate over a very wide range of voltage and frequency, and the voltage and frequency at which the transformer is designed to operate is, therefore, specified. The transformer is rated at a given watts capacity, but the secondary connected load may be less than this. It will, therefore, be of interest to study the variation in voltage ratio and phase angle with secondary load for a given transformer.

Assume the constants of a potential transformer to

be as follows:--

Capacity - 15 watts

$E_1$ = 2200 volts

$N_1$ = 6880 turns

$N_2$ = 344 turns

$\frac{N_1}{N_2} = 20$

$r_1$ = 1000 ohms

$r_2$ = 6. ohms

M = 20 henries

$X_M$ = 7540 ohms at 60 cycles

$L_1'$ = 12 henries (for primary)

$L_1''$ = .03 henries (for secondary)

$x_1$ = 4525 ohms at 60 cycles

$x_2$ = 11.3 ohms at 60 cycles

$R_c$ = 800,000 ohms (primary)

The secondary load is assumed to be non-inductive, as is practically always the case. Then X = 0, and R is the only remaining variable

$$A = R^2 + 6\ R$$

$$B = -\ 11.3\ R$$

$$C = -\ \frac{11.3\ R}{7540} - \frac{20}{800,000}\ (R^2 + 6\ R) - \frac{R}{20}$$

$$D = \frac{(R^2 + 6\ R)}{7540} - \frac{20 \times 11.3\ R}{800,000}$$

$$G = R^2 + 6\ R + \frac{1000}{20}\left[\frac{11.3\ R}{7540} + \frac{20\ (R^2 + 6\ R)}{866,000}\right.$$

$$+ \frac{R}{20}\Big] + \frac{4525}{20}\left[\frac{(R^2 + 6\ R)}{7540} - \frac{20 \times 11.3\ R}{800{,}000}\right]$$

$$H = -\ 11.3\ R - \frac{4525}{20}\left[\frac{11.3\ R}{7540} + \frac{20\ (R^2 + 6\ R)}{800{,}000} + \frac{R}{20}\right]$$

$$+ \frac{1000}{20}\left[\frac{(R^2 + 6\ R)}{7540} - \frac{20 \times 11.3\ R}{800{,}000}\right]$$

Reducing

$$G = R\ (1.03122\ R + 8.6986)$$

$$H = R\ (.000970\ R - 22.9544)$$

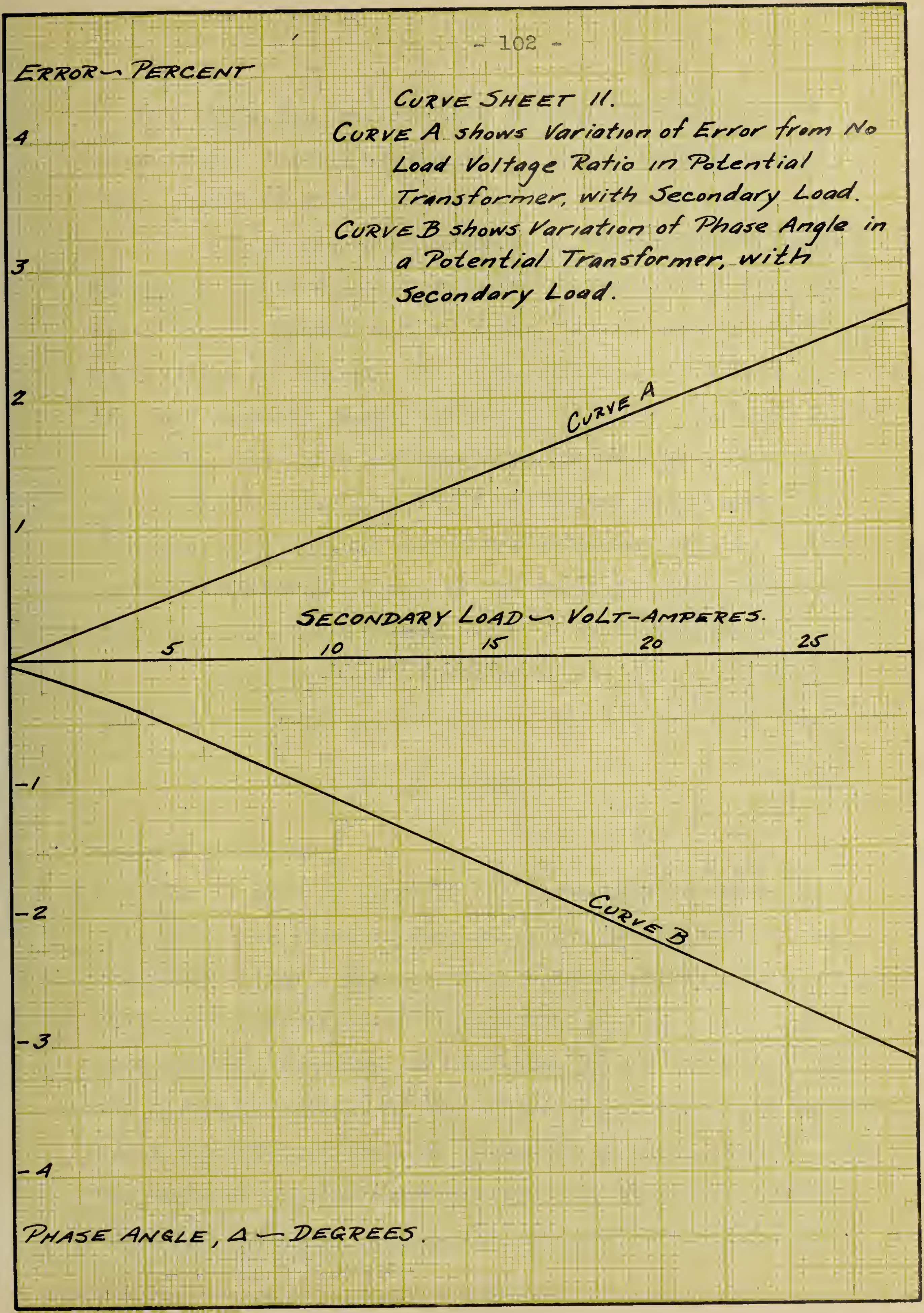
Substituting in (30) and reducing, the voltage ratio is

$$\frac{E_2}{E_1} = -\ \frac{.05\ R}{\sqrt{(1.031\ R + 8.699)^2 + (.00097\ R - 22.95)^2}}$$

and substituting in (31) and reducing, the phase angle is

$$\Delta = \text{arc tan}\ \frac{.00097\ R - 22.95}{1.0312\ R + 8.699}$$

With no load on the secondary, $R = \infty$ , and the voltage ratio is .0485. When secondary load is added, the voltage ratio decreases in practically direct proportion to the load, so that a curve between percent error, as calculated from the no load ratio, and secondary load is practically a straight line thru the origin as shown on Curve Sheet 11, Curve A. With no load on the secondary, the phase angle, $\Delta$ is 3', and increases with the secondary load, the rate of increase in phase angle being slightly greater than the rate of increase in the load, as shown on Curve Sheet 11, Curve B. At the full rated load of 15 watts, the phase angle is about


ERROR — PERCENT
CURVE SHEET 11.
CURVE A shows Variation of Error from No Load Voltage Ratio in Potential Transformer, with Secondary Load.
CURVE B shows Variation of Phase Angle in a Potential Transformer, with Secondary Load.
4
3
2
1
-1
-2
-3
-4
CURVE A
CURVE B
SECONDARY LOAD — VOLT-AMPERES.
5
10
15
20
25
PHASE ANGLE, Δ — DEGREES.

1° 40', while the percent error in ratio from no load is about 1.4%. The potential transformer to be most accurate then, should be calibrated for given secondary load.

It is the usual practice in high potential systems to employ instrument transformers in connection with a wattmeter for the measurement of power, the arrangement being as illustrated in Fig. 25. $E_1$ is the impressed voltage, $I_1$ is the load current, flowing thru the series transformer, R and X are the resistance and reactance of the load respectively, $I_2$ is the secondary current of the series transformer and flows thru the current coilof the wattmeter, and $E_2$ is the secondary terminal voltage of the potential transformer, impressed upon the potential circuit of the wattmeter.

The true power consumed by the load, including the loss in the series transformer and its secondary load, is

$$W = E_1 \, I_1 \cos \theta \qquad (32)$$

where $\theta = \text{arc tan } \frac{X}{R}$

A vector diagram of currents and electromotive forces in this system is given in Fig. 26, where $E_1$ is taken as the reference vector. From previous considerations, the secondary voltage of the potential transformer, $E_2$, generally lags behind the 180° position, as indicated by the angle $\Delta$ , and the secondary current of the series transformer $I_2$ generally leads the 180° position, as indicated by the angle $\beta$ . If the angle $\beta$ be considered positive, as given by equation (19), and the angle $\Delta$ negative, as given by equation (31),

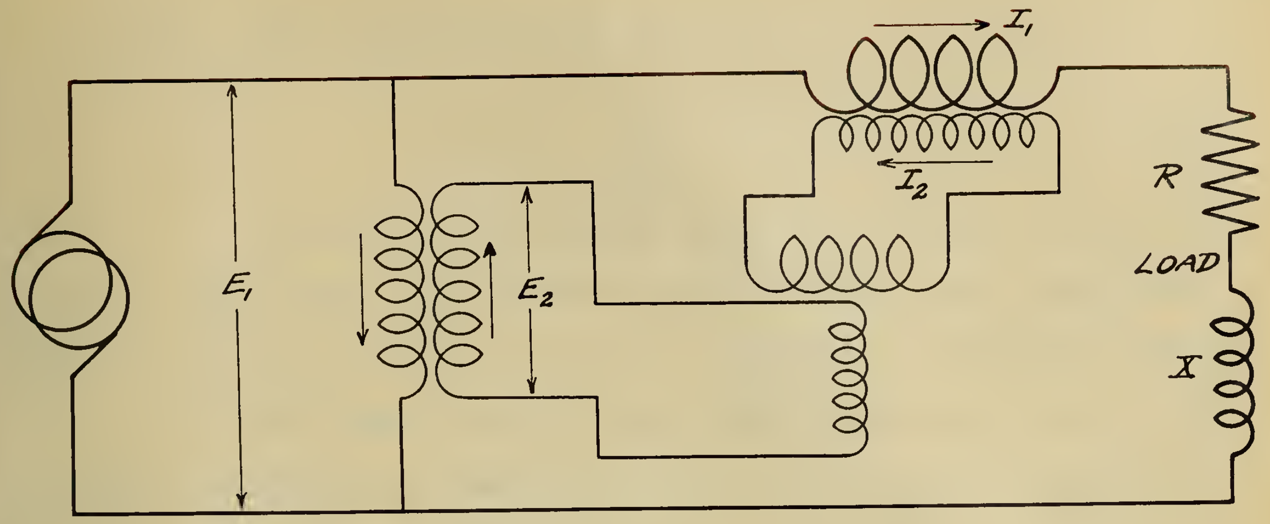



FIG. 25

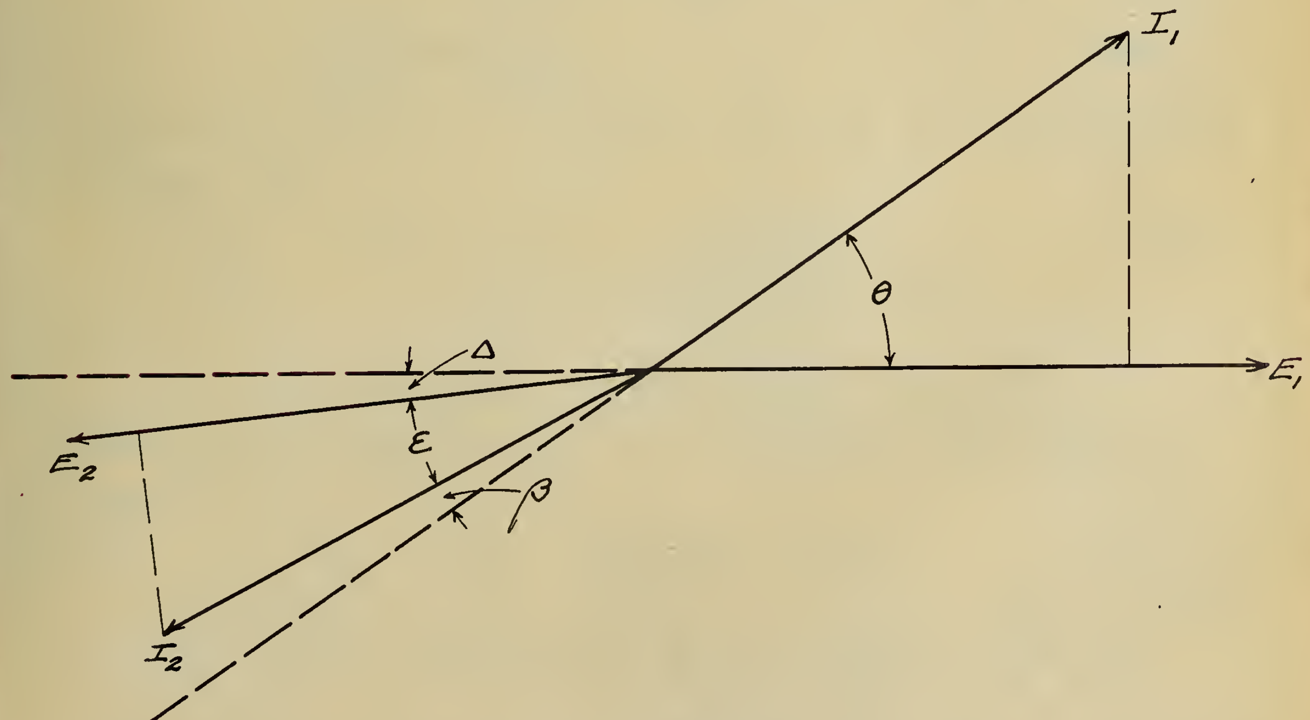



FIG. 26

then the angle $\mathcal{E}$ between $E_2$ and $I_2$ is

$$\mathcal{E} = \theta - (\beta - \Delta) \qquad (33)$$

It will be remembered that at ordinary frequencies and a reasonably high voltage (say 100 volts) the lag angle of the current in the potential circuit of a commercial wattmeter is very small, being less than 1' in the 75 volt instrument considered in Example 1. In connection with instrument transformers this small angle may be neglected and we may say without very great error that the reading of the wattmeter is

$$R = E_2 I_2 \cos \mathcal{E}$$

$$= E_2 I_2 \cos \left[\theta - (\beta - \Delta)\right] \qquad (34)$$

To determine the true power, then, the wattmeter reading is multiplied by a factor obtained by dividing (32) by (34), or

$$k = \frac{E_1 I_1}{E_2 I_2} \; \frac{\cos \theta}{\cos \left[\theta - (\beta - \Delta)\right]} \qquad (35)$$

By trigonometric expansion and reduction by substituting equations (18), (19), (30) and (31), equation (35) becomes:

$$k = \frac{N_{1_P} N_{2_s}}{N_{2_P} N_{1_s}} \; \frac{(A_s^2 + B_s^2)(G_P^2 + H_P^2)}{r^2 \left(A_a G_P + B_s H_P + \frac{X}{R} B_s G_P - \frac{X}{R} A_s H_P\right)} \qquad (36)$$

where the subscripts P and s refer to the potential and series transformers respectively, r is the resistance of the potential circuit of the wattmeter, the reactance having been neglected

as above suggested, and X and R are the reactance and resistance respectively of the load. If the permeability of the series transformer core be assumed constant, and the value of r is fixed, then all the factors in (36) are constant except X and R which appear as the ratio $\frac{X}{R}$. This ratio depends not upon the load, but merely upon the power-factor of the load circuit, and is

$$\frac{X}{R} = \frac{\sqrt{1 - \overline{P.F.}^2}}{P.\ F.}$$

where P. F. represents the power-factor. Hence, under this ideal condition, the factor k depends not upon the load, but upon the power-factor of the load.

Now, if the phase relations between secondary and primary currents and e.m.f.s be not taken into account, but merely the ratio of their magnitudes, as is often the case, then the wattmeter reading is

$$R' = E_2\ I_2 \cos\theta \tag{37}$$

and the calibration factor, k', is given by dividing (32) by (37), or

$$k' = \frac{E_1\ I_1}{E_2\ I_2} \tag{38}$$

which by the substitution of equations (18), (19), (30), and (31) becomes

$$k' = \frac{N_{1_P}\ N_{2_S}}{N_{2_P}\ N_{1_S}}\ \frac{\sqrt{(G_P{}^2 + H_P{}^2)(A_S{}^2 + B_S{}^2)}}{r^2} \tag{39}$$

where the same notation is used as in (36). The value of power obtained by using k' is generally greater than would be obtained by using k because $\varepsilon$ is generally less than $\theta$. The percent error introduced by using k' instead k is

$$e = \frac{k' - k}{k} = \frac{k'}{k} - 1 \tag{40}$$

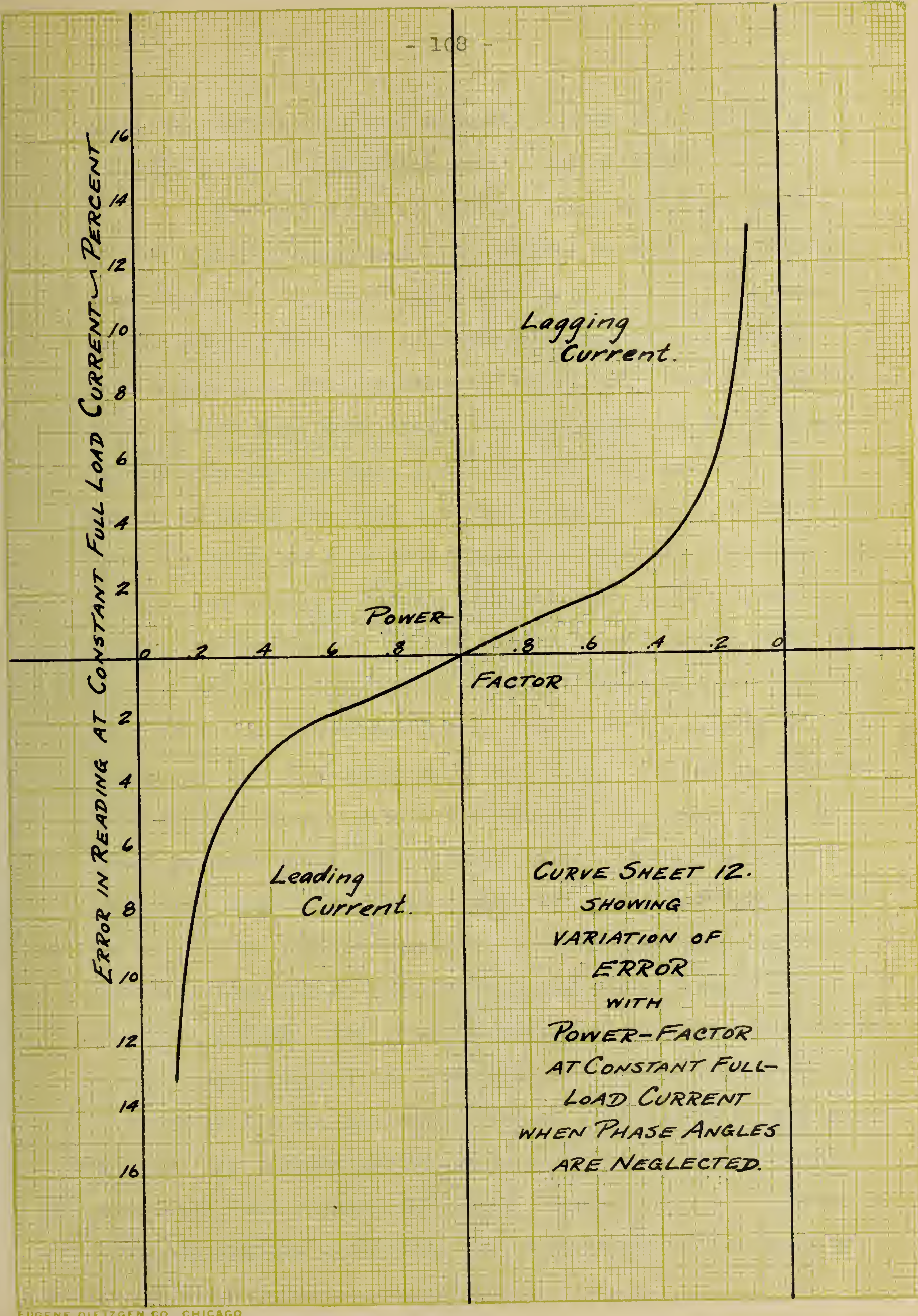
Substituting equations (39) and (36) in (40), and reducing, it becomes

$$e = \frac{\left(A_s G_p + B_s H_p + \frac{X}{R} B_s G_p - \frac{X}{R} A_s H_p\right)}{\sqrt{(A_s^2 + B_s^2)(G_p^2 + H_p^2)}} - 1 \tag{41}$$

or

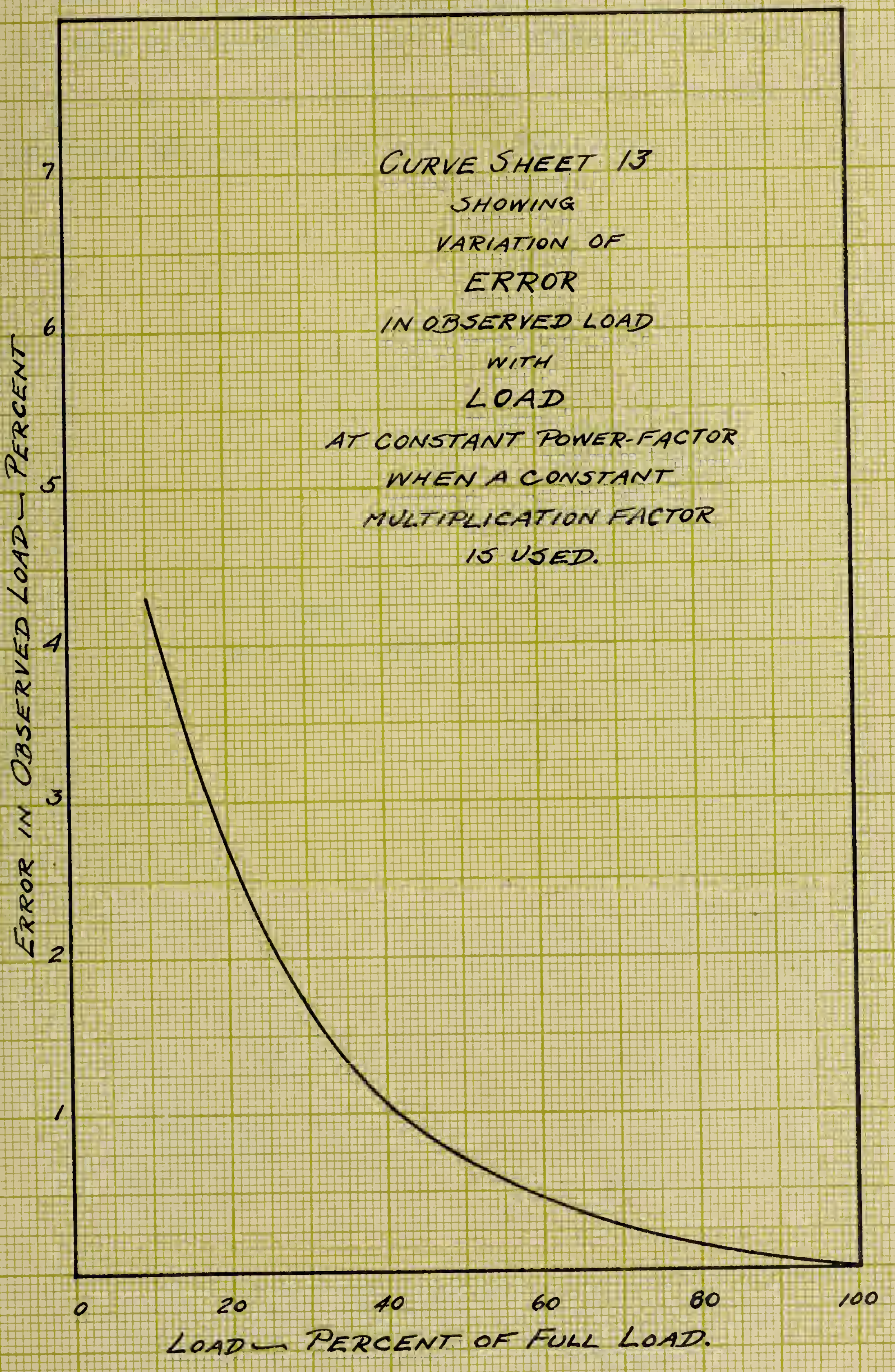
$$e = \frac{(A_s G_p + B_s H_p) + (B_s G_p - A_s H_p)\frac{\sqrt{1 - \overline{P.F.}^2}}{P.F.}}{\sqrt{(A_s^2 + B_s^2)(G_p^2 + H_p^2)}} - 1 \tag{41'}$$

A positive value for e means that the power obtained by using k' is greater than the true value of power, and a negative value means that it is less than the true value. By substituting different values of P. F. in (41') the corresponding percents of error may be found, and thru the points thus determined a curve may be drawn. Such a curve, showing the variation in percent error with the power factor of the load is given on Curve Sheet 12. The percent error for leading power-factors is but so slightly in excess of the percent error for the corresponding lagging power-factor, as to be inappreciable when plotted, but is of opposite sign. At unity power-factor, the percent error is practically


Error in Reading at Constant Full Load Current ~ Percent
16
14
12
10
8
6
4
2
2
4
6
8
10
12
14
16
0
.2
.4
.6
.8
.8
.6
.4
.2
0
Power
Factor
Lagging Current.
Leading Current.
Curve Sheet 12.
Showing
Variation of
Error
with
Power-Factor
at Constant Full-
Load Current
when Phase Angles
are Neglected.

zero, increasing at about a constant rate to .5 P. F. and increasing very rapidly for lower power-factors.

The variation in current ratio and phase angle of the series transformer with primary current also produce a variation in the value of k with load current, the general tendency being for k to increase with decreasing current in such a way that the variation of error with load at constant power-factor will have about the form shown on Curve Sheet 13. The per cent error in this case may be considered negative for the true power is less than would be obtained by using the full load value of k. If in a particular case the value of k is determined for full non-inductive load and the variations given by Curves 12 and 13 be assumed to apply, the conditions for maximum error may be studied. If the current be maintained constant at the full non- inductive load value and the power-factor decreased by lagging current to say.3, then the error will be + 4.3%. If now the current be decreased then a negative error will be added, and the total percent error will decrease until above ten percent full load current is reached when the error becomes practically zero. For lagging currents, then, about ten percent of the full load value, and above .3 P.F., the maximum percent error is ± 4.3%. For leading currents, the percent error is always negative, and for any point of operation is equal to the sum of the two percents obtained from Curves 12 and 13, so that the maximum percent error at .3 P. F. and ten percent full load current is - 8.6%. These would, of course, be quite extreme conditions on a high tension distribution


CURVE SHEET 13
SHOWING
VARIATION OF
ERROR
IN OBSERVED LOAD
WITH
LOAD
AT CONSTANT POWER-FACTOR
WHEN A CONSTANT
MULTIPLICATION FACTOR
IS USED.
ERROR IN OBSERVED LOAD — PERCENT
1
2
3
4
5
6
7
0
20
40
60
80
100
LOAD — PERCENT OF FULL LOAD.

system. The power-factor on a feeder is generally lagging, and very seldom falls below .8. Since in this case the two errors are of opposite sign, the limiting maximum is given by Curve 13, and at ten percent full load current is - 4.3%.

## CONCLUSIONS

A resume of the foregoing discussion must necessatily be very general and brief, for many phases not strictly correlated except thru the very general subject of electrical metering, have been dealt with, and a complete summary would require almost as much space as the discussion itself. It is evident that no electrical metering device is free from error, and in some commercial instruments under certain conditions of operation these errors assume considerable proportions.

In D. C. instruments the chief source of error, assuming constancy of instrument parts and absence of external fields, is temperature changes. For high range voltmeters where a high value of manganin resistance may be employed, this error is generally not very serious, but in millivoltmeters and ammeters employing shunts some form of compensation is necessary to give an accuracy closer than .4% per degree Centigrade variation in temperature.

Alternating current ammeters take all the current thru the instrument and are not, therefore, effected by temperature changes except as these affect the magnetic properties of the iron, nor are their indications seriously affected by frequency or reasonable variations in wave shape. A.C. voltmeters are, however, susceptible to changes in temperature, frequency, and wave shape, such changes being more effective the lower the range of the instrument.

The commercial indicating wattmeter, especially for the higher voltage ranges and when used on reasonably high power-factors is a very accurate instrument, the error in a given example at 2000 cycles and .8 power-factor being less than .25%. When used on power-factors lower than .6, however, the increase in percent error is very rapid, being greater for leading currents than for lagging currents. When used on commercial frequencies, the error is not extremely large even at low power-factors, being less than .2% for .1 lagging power-factor at 60 cycles in the 75 volt meter considered. The lower the voltage rating of the meter the greater the percent error that may be expected, the percent error varying about inversely as the voltage rating. Of course, in a non-compensated meter, the correction for power loss in the meter must be made, which for the potential circuit connected inside the series coil generally ammounts to about 1 watt per 20 volts for full impressed voltage.

Instrument transformers are subject to change in ratio and phase angle with any change in their conditions of operation. Hence the indications of a wattmeter connected to their secondaries are subject to variation with load. On distribution systems where the current is generally lagging and the power-factor seldom falls below .8, the maximum error occurs for low values of current at unity power-factor. At ten percent full load current the error may be as great as 5%.

In conclusion, errors in the indications of meters,

especially under unusual operating conditions, are always to be looked for, and while their magnitude may be quite considerable, they may generally be corrected for by properly taking into account the constants of the instruments.